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CONTRIBUTED PAPER

Exploring fluctuations and interconnected movements in stock, commodity, and cryptocurrency markets

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Abstract

This research employs a vector autoregression (VAR) analysis to explore the volatility and dynamic interactions between stock, commodity, and cryptocurrency markets. It focuses on the returns of the S&P 500, gold, crude oil, and Bitcoin to analyse their interconnections. Our results indicate that Bitcoin returns positively affect S&P 500 and crude oil, but negatively impact gold. Conversely, crude oil returns have a positive influence on gold but lead to decreased returns for Bitcoin and the S&P 500. Similarly, higher gold returns correspond to increased returns in crude oil and S&P 500 but decreased returns in Bitcoin. The rise of the S&P 500 negatively influences Bitcoin and crude oil returns, while gold returns remain unaffected. However, these relationships exhibit weak and limited strength. Including these assets in a portfolio can help risk mitigation, as Bitcoin diversifies crude oil, gold, and S&P 500, and crude oil diversifies S&P 500. These findings contribute to our understanding of global financial dynamics and inform decision-making in risk assessment, portfolio management, risk mitigation, and diversification strategies.

Keywords: Bitcoin return; gold return; oil return; S&P 500 return; vector autoregression (VAR) analysis

1. Introduction

The examination of financial markets and their complex interconnections has always been significant and fascinated the attention of investors, researchers, and policymakers for an extended period. In recent years, the rise of cryptocurrencies, such as Bitcoin, has added a new dimension to this complex landscape, introducing a novel asset class with its own unique characteristics and dynamics (Kruckeberg & Scholz, 2019). Moreover, the traditional financial markets, characterised by S&P 500 stock market and the commodity market encompassing crude oil and gold, maintain their fundamental significance in the global economy and investment approaches (Yang & Cheng, 2014). The unique characteristics and market dynamics of these assets make them intriguing subjects of study. Bitcoin, an independent digital currency, has gathered considerable interest due to its remarkable price fluctuations and its perceived value as a means of preserving wealth or facilitating transactions (Tiwari & Sahadudheen, 2015). Crude oil, a crucial energy resource, holds significant economic implications as its price fluctuations impact global markets and influence inflationary pressures (Ciner, 2013). Based on the research conducted by Choi and Shin (2022), gold, commonly recognised as a safe-haven investment, has traditionally been in high demand during periods of uncertainty. It acts as a protection against inflation and the devaluation of currency. The S&P 500, a widely recognised benchmark index comprising leading US companies, reflects the overall health of the stock market and acts as an indicator of broader economic trends (Yang & Cheng, 2014; Sim & Zhou, 2015).

Understanding the interactions between these different markets has become increasingly important as investors seek to navigate the complexities of the financial world. Moreover, the recent surge in interest and investment in cryptocurrencies has sparked debates about their role in portfolio diversification and risk management (Guesmi *et al.*, 2019). Investigating the connections between the stock market, commodity market, and cryptocurrency can yield valuable insights for optimising portfolios and developing strategies to minimise risk. Nevertheless, despite the increasing attention given to these subjects, there is still a research gap in comprehending the volatility and dynamic relationships among these specific assets, namely Bitcoin, crude oil, gold, and S&P 500. While previous studies (Dyhrberg, 2016; Wang *et al.*, 2016; Miyazaki, 2019; Owusu *et al.*, 2020; Bouri *et al.*, 2020) have individually examined the volatility patterns and interrelationships of these assets, a comprehensive analysis that integrates all of them using vector autoregression (VAR) analysis is lacking. This research gap presents an opportunity to bridge the existing knowledge and contribute to a deeper understanding of the interplay between these markets. By employing a VAR framework, this study aims to address several key questions. First, it seeks to analyse the volatility patterns of each asset individually, exploring their inherent characteristics and identifying any specific factors that drive their price movements. Second, it aims to investigate the existence of spillover effects and dynamic linkages between the assets, assessing whether shocks in one market propagate and impact others. Third, it seeks to uncover potential asymmetries in these relationships, as certain assets may exhibit different responses to positive and negative shocks.

This study's primary objective is to investigate the relationship between stock market, commodities market, and cryptocurrency market, with an emphasis on Bitcoin, crude oil, gold, and S&P 500. By employing VAR analysis, this research aims to provide insights into the short-term and long-term relationships among these assets, quantifying the spillover effects and the impact of shocks. Such analysis can shed light on the potential transmission channels between these markets and contribute to a better understanding of their dynamic interdependencies. Addressing the research questions of whether there is an interaction between the stock market, commodity market, and cryptocurrency, and whether portfolio risk can be minimised through diversification by investing in Bitcoin, crude oil, gold, and S&P 500 is vital in today's complex financial landscape. The findings can guide investors in constructing well-diversified portfolios, taking into consideration the interactions and interdependencies among these assets. Furthermore, policymakers can benefit from understanding the potential risks and stability implications that arise from the integration of these markets.

The selection of Bitcoin, crude oil, gold, and the S&P 500 as the focus of our study is based on several key considerations that align with our research objectives and the dynamics of global financial markets. The first reason is their significance and market influence. Each of the chosen assets holds significant importance in the global financial landscape. These four assets are among the most widely recognised and influential assets in their respective markets. They serve as key indicators of market sentiment, economic trends, and investor behaviour (Ozturk, 2020; Baranovskyi *et al.*, 2021; Hung *et al.*, 2022). By analysing these assets, the aim is to capture a diverse range of market dynamics and interactions. The second reason is their diversification potential. Bitcoin, crude oil, gold, and the S&P 500 offer distinct risk-return profiles and are often considered as potential components of diversified investment portfolios (Guesmi *et al.*, 2019; Jin *et al.*, 2022; Turki *et al.*, 2022). Understanding their interconnections can provide insights into portfolio diversification strategies and risk management practices. The third reason is data availability and research interest. These assets have been the subject of extensive research in finance and economics due to their significant impact on investment decisions, risk management, and macroeconomic trends (Bouri *et al.*, 2020; Choi & Shin, 2022; Nguyen, 2022). By focusing on well-studied assets, it can leverage existing literature and build upon previous findings to contribute new insights to the field. The fourth reason is practical relevance and applications. By focusing on assets that are commonly traded and widely followed by investors, the findings of the study are more likely to have practical relevance and applicability in real-world investment contexts (Hung *et al.*, 2022; Jin *et al.*, 2022). Investors and policymakers can directly benefit from insights into the interactions and dynamics of these assets in their decision-making processes.

2. Literature Review

Following the 2008 financial crisis, there has been an increased requirement to understand the complicated interactions between the stock market, the commodity market and now cryptocurrency. In recent years, crude oil prices have exhibited significant co-movement and exerted considerable influence on economies. Shen *et al.* (2018) observe that crude oil, being a commodity with characteristics of both general and financial commodities, has the potential to trigger heightened volatility and risk contagion in other markets through price fluctuations. Bitcoin, crude oil, gold, and S&P 500 are all popular financial assets that are commonly traded by investors (Hung, 2022). Each of these assets has its own characteristics that can impact their volatility in the marketplace and possibly returns (Engle, 2004). In essence, Bitcoin is a digital asset recognised for its notable volatility (Bakar & Rosbi, 2017), while crude oil is a tangible commodity extensively utilised in transportation and energy generation (Hung, 2022). Gold, on the other hand, is a precious metal frequently employed as a means of preserving value and safeguarding against inflation (Conlon *et al.*, 2018). Lastly, the S&P 500 serves as a stock market index widely employed as a reference point for evaluating the performance of the US stock market (Shen *et al.*, 2018). In general, gold, stocks, crude oil, and then relatively new entrant Bitcoin are likely to serve as safe asset havens for investors seeking to hedge market risks and mitigate the effects of volatility. Based on this knowledge, the current study seeks to understand more about the volatility and dynamic interactions of the returns on Bitcoin, crude oil, gold, and S&P 500.

According to the suggestions of Guesmi *et al.* (2019) and Shahzad *et al.* (2020), Bitcoin exhibits characteristics similar to digital gold, potentially serving as a refuge against downside risks in the stock market. However, Gozbasi *et al.* (2021) discovered that Bitcoin functions as a diversified asset in stable market conditions, but exhibits a positive correlation with the S&P 500 index during volatile periods, indicating its limitations as a reliable safe haven during times of crisis. According to Borri (2019), while Bitcoin poses a higher risk than other cryptocurrencies, it still carries less risk compared to conventional financial assets like the US stock market or gold. This implies that including cryptocurrencies in investment portfolios may provide appealing returns and hedging opportunities. Dyhrberg (2016) proposes Bitcoin as a potential hedge against stocks, whereas Bouri *et al.* (2020) and Shahzad *et al.* (2020) highlight its capacity as a hedging instrument with safe-haven characteristics, potentially surpassing commodities such as gold and oil. Le and Chang (2012) focused on the variability and short-term dynamics of the relationship, while Tiwari and Sahadudheen (2015) examined the effects of real oil price changes on gold and the asymmetric nature of gold price shocks.

According to the research conducted by Le and Chang (2012), there is empirical evidence indicating that oil price shocks have a statistically significant and positive impact on real gold returns at the same time, exhibiting a non-linear and symmetric effect. They highlighted that oil price shocks contribute more to the variability in gold returns than global industrial production but less than other variables in the baseline model. Furthermore, their findings indicated that there is no stable, linear relationship between oil and gold prices. However, Tiwari and Sahadudheen (2015) found that an increase in real oil prices has a positive impact on gold prices. They also observed an asymmetrical effect of shocks on gold prices. Moreover, Kumar (2017) expands on this topic in the Indian context by examining the causal relationship between oil and gold prices. The research emphasises a non-linear and bidirectional causality, with positive shocks in oil prices exerting a more significant influence than negative shocks in gold prices.

The studies by Ciner (2013), Miyazaki (2019), and Sim and Zhou (2015) provide valuable insights into the complex relationship between oil prices, stock returns, and gold returns. Ciner's

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(2013) research emphasises the substantial influence of oil price shocks on stock market indexes, demonstrating varied reactions that can be either negative or positive depending on the duration of the shocks. This implies that the impact of oil price shocks on stocks is non-linear and fluctuates depending on the length of the shocks. This suggests that the effects of oil price shocks on stocks are non-linear and vary based on the duration of the shocks. In contrast, Miyazaki (2019) explores the relationship between gold returns and various economic factors, emphasising the negative correlation between gold and stock returns. The asymmetric response of gold returns to stock market volatility and financial market stress underscores gold's role as a safe haven asset during periods of market turmoil. Additionally, Sim and Zhou (2015) shed light on the asymmetric relationship between oil prices and US equities, with negative oil price shocks exerting a positive influence on equities during favourable market conditions. This discussion collectively suggests that the dynamics between oil prices, stock returns, and gold returns are complex and multifaceted, influenced by factors such as shock persistence, market conditions, and economic uncertainty. Furthermore, Mamipour and Vaezi Jezeie (2015) focus on the Iranian context, demonstrating the positive short-term impact of oil prices on stock returns but a negative long-term relationship.

Panagiotidis *et al.* (2018) and Nguyen (2022) conducted studies investigating the drivers of Bitcoin returns and the association between Bitcoin and the stock market during periods of elevated uncertainty. Panagiotidis *et al.* (2018) examined various factors and identified search intensity, gold returns, and policy uncertainty as the primary determinants influencing Bitcoin returns. Their study observed both positive and negative relationships between Bitcoin returns and factors such as exchange rates, interest rates, gold, oil, and stock markets. However, the impact of these variables varied depending on the specific factor and time period under analysis. In contrast, Nguyen (2022) focused on the influence of the stock market on Bitcoin returns, particularly during the COVID-19 pandemic. Their findings revealed a significant relationship between the returns of the S&P 500 and Bitcoin, indicating an increased correlation between the stock market and cryptocurrency during times of crisis. Jareno *et al.* (2020) employ quantile regression analysis to identify significant determinants of Bitcoin returns, highlighting the negative impact of the VIX (American Stock Market) index and STLFSI (Saint Louis Financial Stress Index) on Bitcoin returns. Furthermore, the study establishes a positive relationship between Bitcoin and gold price returns, suggesting Bitcoin's potential as a safe-haven asset during economic uncertainty. In contrast, Liu and Naktnasukanjn (2022) investigate the dynamic correlation between Bitcoin, crude oil, and gold, offering insights into their risk profiles and correlation patterns. Their findings reveal Bitcoin's high risk, the lack of significant correlation between gold and crude oil, and the changing correlation dynamics between Bitcoin and the two commodities during the COVID-19 pandemic.

The relationship between Bitcoin, gold, and other financial assets, is examined including their roles as speculative assets, safe havens, hedges, and diversified by Zwick and Syed (2019), Kyriazis (2020), Drake (2022), and Owusu *et al.* (2020). Zwick and Syed (2019) find a non-linear relationship between Bitcoin and gold, with a structural break indicating Bitcoin's transition from a speculative asset to a diversifier and hedge after October 2017. However, the impact of gold on Bitcoin prices is found to be non-linear over the studied period. Kyriazis (2020) summarises empirical studies and highlights the mixed findings regarding Bitcoin's status as a safe haven and its relationship with gold. The literature suggests that Bitcoin's characteristics as a safe-haven asset are still developing and that gold may offer better hedging properties against Bitcoin. Drake (2022) challenges the traditional belief that gold is a safe haven asset with a negative correlation to stock returns, presenting evidence that during periods of negative real rates of return, the gold-stock market relationship becomes positive, indicating gold's safe haven role during stock market volatility and negative interest rates. Lastly, Owusu *et al.* (2020) explore the hedging and diversification potentials of gold and cryptocurrencies, finding that both assets can hedge and diversify each other depending on conditional distributions and that cryptocurrencies are influenced by medium- and long-term fundamentals. This means that the returns on these types of financial assets will likely exhibit dynamic interactions. For example, during times of economic uncertainty, investors may shift investments in stocks to safe-haven assets like gold, which can cause the price of gold to increase while the S&P 500 would likely decrease (Jones & Sackley, 2016). Similarly, changes in global supply and demand for crude oil can impact the prices of both crude oil and the S&P 500 (Hung, 2022), as fluctuations in oil prices will likely have a "ripple effect" throughout economies. Understanding the volatility and dynamic interactions of these financial assets is important, especially for investors who are seeking to manage their risk and optimise their investment portfolios. In actual fact, by analysing historical data and monitoring current market conditions, investors can gain valuable insights into the behaviour of these assets and are able to make more informed investment decisions.

A growing literature has considered the nature of the instability and dynamic interactions of these financial assets. Shahzad and colleagues (2020) have focused on comprehending the intricate distinction between gold and Bitcoin in relation to G7 stock markets. Their findings reveal that gold and Bitcoin possess unique qualities as safe havens and hedging instruments. Gold demonstrates significantly higher hedging effectiveness compared to Bitcoin. This growing body of literature aims to uncover the interdependencies, correlations, and spillover effects among these assets. By investigating the interconnectedness of these markets, Shahzad and colleagues (2020), seek to enhance our comprehension of how shocks and changes in one market can propagate and impact others, thereby providing valuable insights for risk management, portfolio diversification, and investment strategies in an increasingly interconnected global economy. Nonetheless, when we consider particular interactions between the stock market, commodity market and cryptocurrency, we find that it is still not possible to categorically say whether there is a clear interaction between the stock market, commodity market and cryptocurrency. Whether portfolio diversification leads to portfolio risk minimisation, by investing across these markets concurrently? This leads us to the main research questions:

- Is there an interaction between the stock market, commodity market and cryptocurrency?
- Can portfolio risk be minimised through diversification by investing in Bitcoin, crude oil, gold, and the S&P 500?

For this reason, we have utilised the VAR model. To forecast and better understand financial time series dynamics, relative to the intricate dynamics within the global financial system, more effectively.

3. Methodology

Bitcoin, crude oil, gold, and the S&P 500 were selected in our study due to a couple of reasons. First, these assets are globally significant and influential, serving as vital indicators of market sentiment and economic trends (Ozturk, 2020; Baranovskyi *et al.*, 2021; Hung *et al.*, 2022). Second, they offer diverse risk-return profiles, making them essential components of diversified investment portfolios (Guesmi *et al.*, 2019; Jin *et al.*, 2022; Turki *et al.*, 2022). Third, their extensive research interest (Bouri *et al.*, 2020; Choi & Shin, 2022; Nguyen, 2022) and data availability provide a rich foundation for analysis, enabling them to build upon existing literature and contribute new insights. Lastly, their practical relevance and widespread use in real-world investment contexts (Hung *et al.*, 2022; Jin *et al.*, 2022) ensure that our findings have direct applicability for investors and policymakers seeking to make informed decisions in the financial markets.

Daily data from 1 January 2015 to 31 March 2023 for Bitcoin price, crude oil price, gold spot price, and S&P500 were sourced from investing.co.uk. The returns are determined by taking the natural logarithm of the ratio between consecutive prices (Katsiampa, 2017). Descriptive statistics and

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graphical analysis were used to describe the variables statistically, and it verified the data appropriateness for econometric analysis. Subsequently, formal pre-tests were carried out to examine the presence of serial correlations, assessed through Collinearity Statistics (Meiryani *et al.*, 2022), as well as the stationarity of the time series using the augmented Dickey–Fuller test (ADF) (Wang *et al.*, 2016).

The VAR model is widely recognised as a highly effective, flexible, and straightforward approach for analysing multivariate time series data. Sims (1980) has a significant role in introducing VAR models to the field of economics by extending the concept of univariate autoregressive models. VAR models are valuable tools for both forecasting and comprehending the dynamic behaviour of financial time series. In fact, VAR models often outperform more complex theory-based simultaneous equations models and univariate time series models in terms of forecast accuracy. Their flexibility stems from the ability to incorporate specific variables of interest and create forecasts that are contingent on various future scenarios outlined within the model. The time series with the VAR model was used in the financial literature (Khalid & Kawai 2003; Hondroyiannis *et al.* 2005; Choi & Shin 2022). The following steps can be applied while creating a VAR model.

While VAR models are commonly used for forecasting, the priority in this study may not be on future predictions but rather on understanding the historical relationships and interactions among the selected assets. The primary focus of our analysis is understanding how shocks or movements in one market affect others, rather than solely on predicting future values. According to Wu and Zhou (2010), VAR models capture both the short-term and long-term interactions among the variables and conduct impulse response analysis, which helps in understanding how a shock to one variable affects the others over time. Also, the VAR model can be used for analysing historical data to uncover patterns and relationships in addition to future predictions (Gregory & Reeves, 2008; Wu & Zhou, 2010). Hence, it helps to understand how different assets move to each other. It assists in investigating portfolio risk minimisation through diversification by revealing the relationship between the variables (Marinescu *et al.*, 2013).

3.1. Vector AR(p) Models

The VAR(p) model is followed by the time series Y_t if it meets the requirements and assumptions of the model:

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + \ldots + \Phi_p Y_{t-p} + a_t, \mathrm{p} > 0 \tag{1}$$

The equation 1 comprises a vector ϕ_0 with k dimensions and a sequence of uncorrelated random vectors a_t, characterised by a mean of zero and a covariance matrix Σ. The positive definiteness of Σ is essential for preserving the dimension of Y_t. The error term a_t follows a multivariate normal distribution, while Φ_j represents k x k matrices. The VAR(p) model can be represented using the back-shift operator B, as depicted below:

$$\left(I - \Phi_1 B - \ldots - \Phi_p B^p\right) Y_t = \phi_0 + a_t$$

The identity matrix I has dimensions of k x k. It can be expressed more simply as follows:

$$\Phi(\mathrm{B}) Y_t = \phi_0 + a_t,$$

The expression $\Phi(\mathrm{B}) = I - \Phi_1 B - \ldots - \Phi_p B^p$ represents a matrix polynomial. If Y_t is characterised by weak stationarity, it can be expressed as follows:

$$\mu = E(Y_t) = \left(I - \Phi_1 - \ldots - \Phi_p\right)^{-1} \phi_0 = [\Phi(1)]^{-1} \phi_0$$

Given that $[\Phi(1)]$'s determinant is not zero, it is dependent on the existence of an inverse.

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Assuming $\widetilde{Y_t} = Y_{t-\mu}$. Subsequently, the VAR(p) form can be expressed as follows:

$$\widetilde{Y_t} = \Phi_1 \widetilde{Y_{t-1}} + \ldots + \Phi_p \widetilde{Y_{t-p}} + a_t \tag{2}$$

By employing the equation 2 provided below, results can be obtained.

- $COV(Y_t,\ a_t) = \Sigma$, the covariance matrix of a_t;
- $COV(Y_{t-1},\ a_t) = 0$ for I > 0,

$$\Gamma_I = \Phi_1 \Gamma_{I-1} + \ldots + \Phi_p \Gamma_{I-p} \text{ for I} > 0 \tag{3}$$

Equation 3, also referred to as the moment equation for a VAR(p) form, is a multivariate variation of the Yule-Walker equation.

3.2. Building a VAR(p) Model

It is possible to determine the ideal order p for a vector series by extending the partial autocorrelation function, which is frequently used to analyse individual time series. In this analysis, we consider a series of consecutive VAR models:

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + a_t$$

$$Y_t = \phi_0 + \Phi_1 Y_{t-1} + \Phi_2 Y_2 + a_t$$

$$\ldots = \ldots$$

$$= \phi_0 + \Phi_1 Y_{t-1} + \ldots + \Phi_i Y_{t-i} + a_t \tag{4}$$

$$\ldots = \ldots$$

The parameters of these models are estimated using the ordinary least squares (OLS) approach, commonly known as multivariate linear regression estimation and frequently used in multivariate statistical analysis (Tsay, 2005).

The OLS estimate of Φ_j, denoted as $\hat{\phi}_j^{(i)}$, and the estimate of $_0$, denoted as $\widehat{\Phi}_j^{(i)}$, are obtained for the i-th equation in Equation 3. The superscript (i) signifies that these estimates are specific to a VAR(i) model. Consequently, the residual can be expressed as:

$$\hat{a}_t^{(i)} = Y_t - \widehat{\Phi}_1^{(i)} Y_{t-1} - \ldots - \widehat{\Phi}_i^{(i)} Y_{t-i}$$

For i = 0, the residual is characterised as:

$$\widehat{Y}_t^{(0)} = Y_t - \overline{Y},$$

where;

$\overline{Y}$ denotes the sample mean of Y_t. The covariance matrix of the residual is defined as follows:

$$\widehat{\Sigma}_i = \frac{1}{T - 2i - 1} \sum\nolimits_{t=i+1}^{T} \hat{a}_t^{(i)} \left(\hat{a}_t^{(i)}\right)^{|} \tag{5}$$

Using the i-th and (i-1)th equations in equation 4, we may compare a VAR(i) model to a VAR(i-1) model and use the results to establish the proper order p. The test aims to evaluate the null hypothesis $H_0: \ \Phi_l = 0$ against the alternative hypothesis $H_a: \ \Phi_l \neq 0$, where the test is conducted sequentially for $\iota = 1, 2, \ldots$ (Box & Tiao, 1977). The test statistic can be calculated as follows:

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$$M(i) = -\left(T - k - i - \frac{3}{2}\right)Ln\left(\frac{\left|\widehat{\Sigma}_i\right|}{|\widehat{\Sigma}_{i-1}|}\right)$$

The chi-squared distribution with k^2 degrees of freedom determine the distribution of M(i). Alternatively, the order p can be determined using the Akaike information criterion (AIC). Assuming that a_t follows a multivariate normal distribution, the i$^{\text{th}}$ equation in Equation 4 can be employed to estimate the model using the maximum likelihood (ML) method. In the case of autoregressive (AR) models, the OLS estimates of ϕ_0 and Φ_j are equivalent to the (conditional) ML estimates. However, the estimates of Σ differ between the two methods. According to Tsay (2005), the ML estimate of Σ is given by:

$$\widehat{\Sigma}_i = \frac{1}{T}\sum\nolimits_{t=i+1}^{T} \hat{a}_t^{(i)}\left(\hat{a}_t^{(i)}\right)^{|} \tag{6}$$

The AIC for a VAR(i) model assuming normality is given by:

$$AIC(i) = Ln\left(\left|\widehat{\sum}_i\right|\right) + \frac{2k^2 i}{T} \tag{7}$$

when working with a vector time series, the order p for the AR model can be selected by finding the positive integer value of p that minimises AIC(p) among all $1 \leq i \leq p$.

3.3. Structural Analysis by Impulse Response Functions

Equation 1 represents the standard structure of the VAR(p) model, which can also be expressed using a Wold representation as:

$$Y_t = \mu + a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \ldots \tag{8}$$

The element at the (i, j)-th position of the matrix θ_s, denoted as θ_{ij}^s, corresponds to the dynamic multiplier or impulse response that indicates the impact of a unit shock in the j-th variable on the i-th variable at time s:

$$\frac{\partial y_{i,t+s}}{\partial a_{j,t}} = \frac{\partial y_{i,t}}{\partial a_{j,t-s}} = \theta_{ij}^s \; i,j = 1,2,\ldots,\mathrm{n} \tag{9}$$

Equation 9 holds true when the variance–covariance matrix of the error term, denoted as Var(a_t) $= \Sigma$, is in a diagonal form. When Σ is diagonal, it means that the elements of the error term a_t are uncorrelated. One approach to achieve uncorrelated errors is to estimate the triangular structural VAR(p) model:

$$Y_{1t} = C_1 + a`_{11} Y_{t-1} + \ldots + a`_{1p} Y_{t-p} + \eta_{1t}$$

$$Y_{2t} = C_1 + \beta_{21} Y_{1t} + a`_{21} Y_{t-1} + \ldots + a`_{2p} Y_{t-p} + \eta_{2t}$$

$$Y_{nt} = C_1 + \beta_{n1} Y_{1t} + \ldots + \beta_{n,n-1} Y_{n-1,t} + a`_{n1} Y_{t-1} + \ldots + a`_{np} Y_{t-p} \eta_{nt} \tag{10}$$

The calculated covariance matrix of the error vector η_t can ensure that the errors are uncorrelated or orthogonal to each other. This means that the elements of η_t do not have any correlation with each other. This is achieved by estimating the triangular structural VAR(p) model. The Wold representation of Y_t which is based on these orthogonal errors η_t, can be expressed as:

$$Y_t = \mu + \Theta_0 \eta_t + \Theta_1 \eta_{t-1} + \Theta_2 \eta_{t-2} + \ldots$$

where the matrix B used in the expression for $\Theta_0 = B^{-1}$ is a lower triangular matrix, where the diagonal elements of B$_{i,j}$ are equal to 1 in equation 10. The impulse responses for the orthogonal

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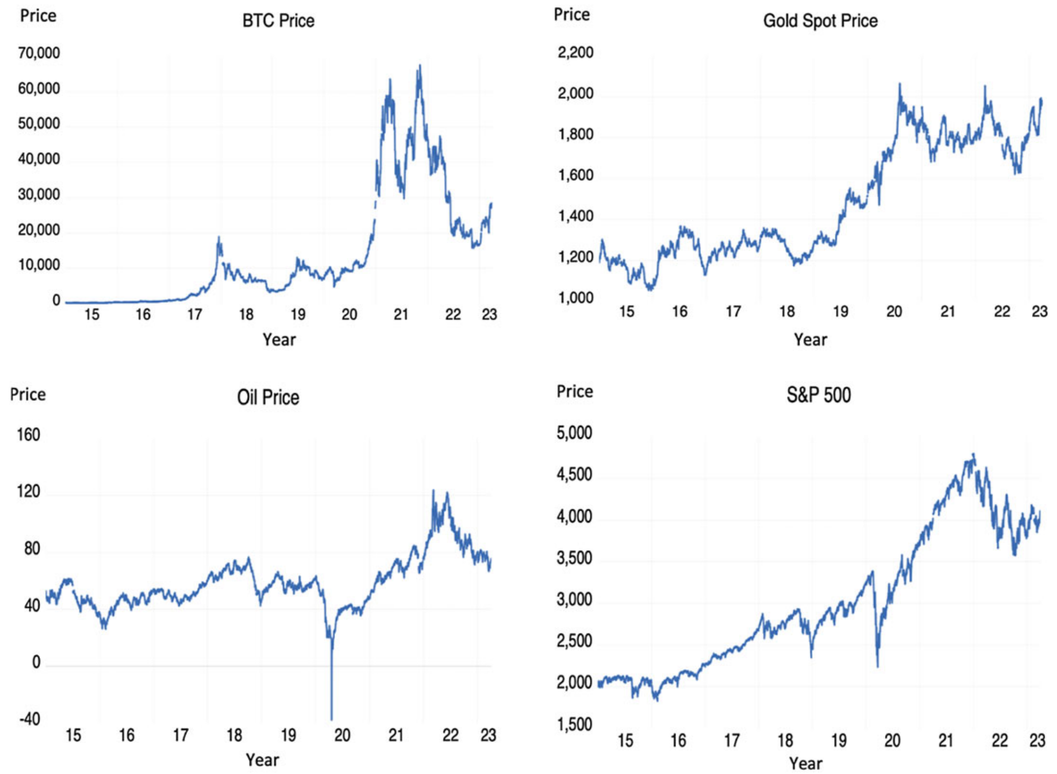

Figure 1. The price volatility of Bitcoin, Gold, Crude Oil, and S&P 500.

shocks η_{jt} can be obtained from the expression provided. $\frac{\partial y_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial y_{i,t}}{\partial \eta_{j,t-s}} = \theta^{s}_{ij}$, where θ^{s}_{ij} is the (i,j) th element of $_s$. The values of θ^{s}_{ij} over time (s) is known as the orthogonal impulse response function of Y_i concerning η_j.

4. Empirical Results

VAR model was employed to understand the volatility of Bitcoin (BTC), Gold, Crude Oil, and S&P 500 returns and dynamic interactions among crypto, stock, and commodity markets.

Figure 1 represents the price volatility of Bitcoin, Gold, Crude Oil, and the S&P 500 for 8 years. The x-axis of each figure demonstrates the years from 2015 to 2023, and the y-axis presents the price volatility for each of the assets in the dollar. Bitcoin, Gold, and S&P 500 have been volatile with significant fluctuations. Oil price volatility has been relatively stable over the years with only minor fluctuations. The oil price dropped to the lowest level in the last 8 years in 2020 due to a couple of reasons. As a result of the pandemic's impact on the world economy, fewer people travelled and engaged in other economic activities, which reduced demand for oil. Producers faced the obligation to pay for the disposal of surplus supply, leading to oil prices plummeting below zero (Guardian, 2020). Additionally, a price war between Russia and Saudi Arabia contributed to this situation, as both countries increased their oil production, causing a decline in oil prices in March 2020 (Ma *et al.*, 2021). Furthermore, Bitcoin exhibits higher price volatility compared to crude oil, gold, and S&P 500.

Figure 2 illustrates the return volatility of Bitcoin, gold, crude oil, and S&P 500 from January 2015 to March 2023. The returns of Bitcoin show considerable volatility, experiencing significant fluctuations over the years compared to other financial assets. Gold, on the other hand, has a

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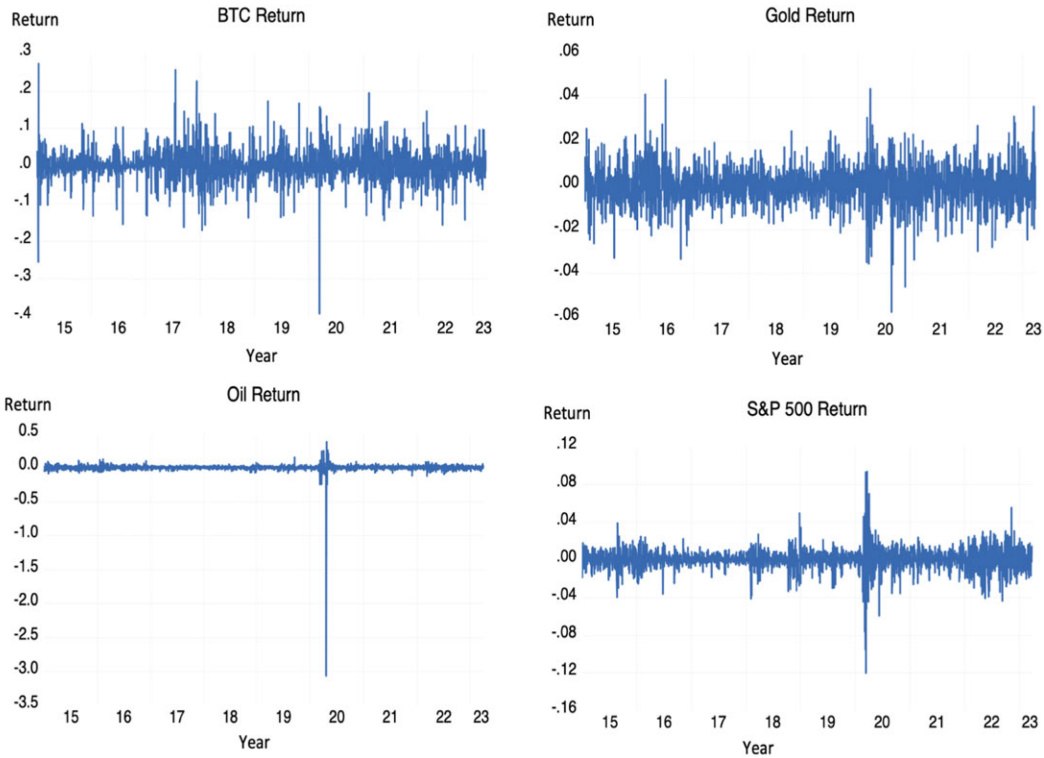

Figure 2. The return volatility of Bitcoin, Gold, Crude Oil, and S&P 500.

reputation as a safe-haven asset, with its price influenced by economic and geopolitical factors such as inflation, interest rates, and global uncertainty. For instance, investors purchased gold as a hedge against market volatility and economic uncertainty during the COVID-19 epidemic, which helped to increase its price (Choi & Shin, 2022). However, compared to other investment alternatives like the stock market or cryptocurrencies like Bitcoin, gold's return during the past 8 years has been rather low. It can be said that the return on crude oil has been relatively stable when the year 2020 is ignored. Despite some periods of volatility, including the COVID-19 pandemic in 2020, S&P 500 has remained resilient over the last 8 years, with many investors continuing to see it as an attractive investment opportunity. Generally, the returns of Gold and S&P 500 have been representing similar patterns.

Figure 3 and Table 1 present the descriptive statistics and histogram of the returns for Bitcoin, Crude Oil, Gold, and S&P 500. Over the 8-year period, the average return for Bitcoin is 0.24%, indicating a positive return. The standard deviation of 0.041838 suggests relatively high volatility in the returns. The minimum value of −39.18% and the maximum value of 27.20% highlight the significant fluctuations in Bitcoin returns during the observed period. The negative skewness value of −0.1974 indicates a slight leftward skew in the data. The kurtosis value of 11.1147 is notably high, indicating heavy tailenders and a more peaked distribution than the normal distribution. This suggests a higher likelihood of extreme values compared to a normal distribution. The Jarque–Bera test, assessing the normality of the data, yields a high value of 5709.403, indicating that the data is not normally distributed and deviates from a normal distribution (Jarque & Bera 1980).

According to Figure 3 and Table 1, the returns on gold are not normally distributed and are more likely to have extreme values than the average return. While the kurtosis (6.1202) shows that the data has a larger likelihood of extreme values, the negative skewness (−0.1267) says that there

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Table 1. Descriptive statistics

	BTC return	Gold return	Oil return	S&P 500 return
Mean	0.002544	0.000273	−0.001248	0.000403
Median	0.001500	0.000500	0.001800	0.000600
Max	0.272000	0.048000	0.376600	0.093800
Min	−0.391800	−0.057300	−3.059700	−0.119800
Std. dev.	0.041838	0.008817	0.079047	0.011840
Skewness	−0.197466	−0.126739	−29.81124	−0.514469
Kurtosis	11.114720	6.120206	1112.851	17.04577
Jarque–Bera	5709.403	847.6948	1.07E+08	17156.62
Probability	0.000000	0.000000	0.000000	0.000000

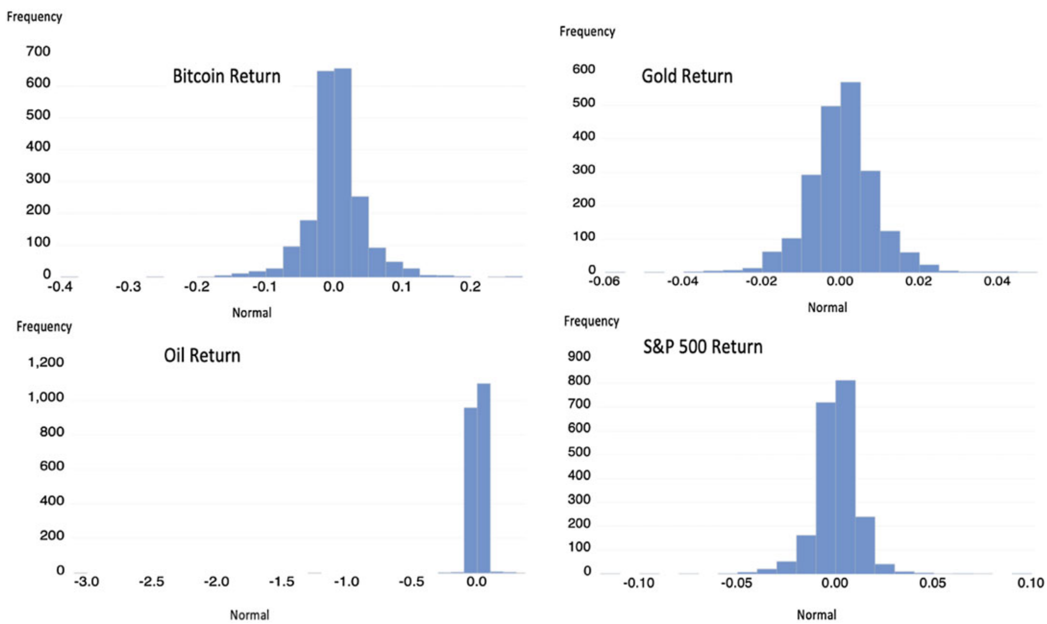

Figure 3. Histogram of Bitcoin, Gold, Crude Oil, and S&P 500 returns.

is a somewhat higher probability of lower returns. The Jarque–Bera test (847.6948) confirms that the returns of gold do not follow a normal distribution. The average return on oil over the provided period is −0.12%, indicating a negative average rate of return, which means that oil's value dropped on average during this time. The extremely positive kurtosis value, which is supported by the kurtosis value of 1112.851, indicates that the distribution of oil returns is highly peaked and has a high likelihood of extreme values. The skewness of −29.8112 indicates a highly negative skewness, which means that the data is highly skewed to the left, with an asymmetric distribution. This suggests that a small number of extremely negative returns are responsible for the skewness measurement. The Jarque–Bera test demonstrates that the distribution of oil returns is not normally distributed. S&P 500 returns are not normally distributed and exhibit a greater likelihood of extreme values compared to the normal distribution. A negative skewness of the data (−0.5144) suggests a slightly higher likelihood of lower returns, whereas a high kurtosis (17.0457)

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Table 2. Pearson correlation matrix

	BTC return	Gold return	Oil return	S&P 500 return
BTC return	1	.098**	.046*	.221**
Gold return	.098**	1	0.023	0.018
Oil return	.046*	0.023	1	.151**
S&P 500 return	.221**	0.018	.151**	1

**Correlation is significant at the 0.01 level (2-tailed).
*Correlation is significant at the 0.05 level (2-tailed).
No of observation: 2076.

implies a higher probability of extreme values. The results of the Jarque–Bera (17156.62) test confirm that the returns of the S&P 500 do not conform to a normal distribution.

Table 2 shows the correlation matrix for the returns of Bitcoin (BTC), Gold, Crude Oil, and the S&P 500. The Pearson correlation coefficient is employed to assess the strength and direction of the linear relationship between these variables. Investors typically seek negatively correlated assets, lower positively correlated assets, or uncorrelated assets to mitigate portfolio risk (Bouri *et al.*, 2020). The results reveal significant correlations between Bitcoin and Gold, Bitcoin and Oil, Bitcoin, and S&P 500, as well as Oil and S&P 500 returns. With a correlation coefficient of 0.098, the relationship between Bitcoin and Gold is significant, positive, and weak. This shows that although there is a weak association between the two variables, there is a modest tendency for them to move in the same direction. The Bitcoin and Oil Return exhibit a significant and positive but weak correlation, as indicated by the correlation coefficient of 0.046. The correlation between Bitcoin returns and S&P 500 returns is significant, positive, and moderately strong, with a correlation coefficient of 0.221. It shows that there is a relatively strong tendency for the two variables to move in the same direction. S&P 500 return and Oil return demonstrate a significant, positive, and moderately strong correlation, as evidenced by the correlation coefficient of 0.151. This suggests that there is a relatively robust inclination for the two variables to change in the same direction. There is no significant correlation for other combinations. The time series' stationary or non-stationary status was determined using the ADF test. It is an essential part of time series analyses to determine trends, conduct forecasting and do regression analyses (Roy *et al.*, 2018).

The returns of Bitcoin, Gold, Oil, and the S&P 500 were subjected to ADF test in order to understand whether there is a seasonality effect in data. The test statistics for each asset are below the threshold values. The null hypothesis of a unit root can be rejected at a 95% confidence level for Bitcoin returns, as the test statistic (−46.86) is more negative than the critical value (−2.86), and the p-value is less than 0.05 (p-value = 0.0001). This indicates that Bitcoin return time series is stationary and does not possess a unit root. Similarly, at a 95% confidence level, the null hypothesis of a unit root in Gold return time series can be rejected, as the test statistic (−44.71) is more negative than the critical value (−2.86), and p-value is less than 0.05 (p-value = 0.0001). This provides evidence that Gold return time series is stationary and does not exhibit a unit root. Furthermore, at a 95% confidence level, the null hypothesis of a unit root in oil return and S&P 500 time series can be rejected, as the test statistics (−33.37 and −14.44) are more negative than the critical value (−2.86), and the p-values are less than 0.05 (p-value = 0.0000). This suggests that oil and S&P 500 return time series are stationary and do not possess a unit root. Consequently, we can reject the null hypothesis of non-stationarity for all four assets at any reasonable significance level, indicating that the returns for these assets are stationary and there is no seasonality effect in the data.

Figure 4 presents the autocorrelations for various financial variables such as Bitcoin return, Gold return, Crude Oil return, and S&P 500 return. Autocorrelation measures the correlation between a variable and its lagged values. The autocorrelations in the document are presented with

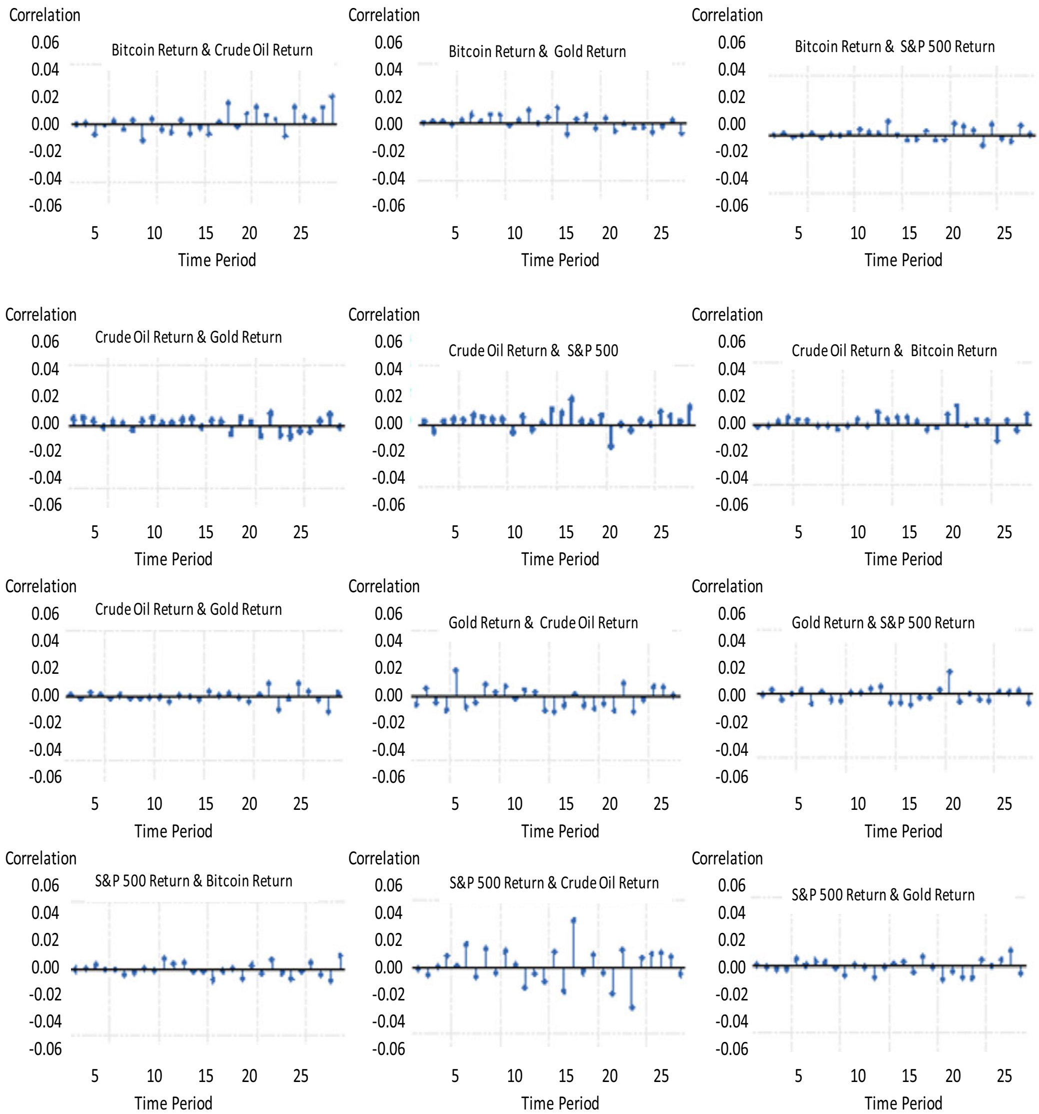

Figure 4. Autocorrelations for Bitcoin, Crude Oil, Gold, and S&P 500.

approximately 2 standard error bounds, which can be used to test the statistical significance of the correlations. ADF test results were supported with autocorrelations, and it can be clearly said that there is no autocorrelation. In a nutshell, ADF test results and correlations indicate that the returns of Bitcoin, Gold, Crude Oil, and S&P 500 are stationary, which implies that their means, variances, and autocovariance do not change over time. Therefore, the VAR model is the appropriate model for analysing the time series (Tsay, 2005).

Table 3 presents the VAR lag order selection criteria, which are used to determine the appropriate lag order for the VAR model. Various criteria are available for this purpose. The AIC is a widely used measure that takes into account both the goodness of fit and the complexity of the model. Lower AIC values indicate better models as they penalise models with more parameters. In this study, the AIC suggests a lag length of 28. Additionally, Schwarz's Bayesian Criterion (SC) and Hannan–Quinn (HQ) criteria are employed to determine the lag length, which indicates a lag

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Table 3. VAR lag order selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	18961.47	NA	1.05E-13	−18.53125	−18.52025	−18.52722
1	19089.92	256.2841	9.43E-14	−18.64118	−18.58620	−18.62101
2	19153.12	125.8374	9.00E-14	−18.68731	−18.58836*	−18.65102*
3	19162.26	18.16756	9.06E-14	−18.68061	−18.53768	−18.62819
4	19180.18	35.54841	9.04E-14	−18.68249	−18.49558	−18.61394
5	19192.2	23.78949	9.08E-14	−18.67860	−18.44771	−18.59392
6	19215.34	45.71342	9.02E-14	−18.68557	−18.41071	−18.58477
7	19249.28	66.90381	8.86E-14	−18.70310	−18.38427	−18.58617
8	19266.36	33.61407	8.85E-14	−18.70416	−18.34135	−18.57110
9	19309.08	83.89612	8.62E-14	−18.73028	−18.32349	−18.27710
10	19322.61	26.51629	8.64E-14	−18.72787	−18.58109	−18.56254
11	19336.94	28.03076	8.66E-14	−18.72623	−18.23149	−18.54478
12	19349.88	25.26778	8.68E-14	−18.72325	−18.18452	−18.52567
13	19374.83	48.59822	8.61E-14	−18.73199	−18.14929	−18.51828
14	19388.99	27.54429	8.62E-14	−18.73020	−18.10352	−18.50036
15	19399.55	20.47411	8.67E-14	−18.72487	−18.05421	−18.47891
16	19416.29	32.43067	8.66E-14	−18.72560	−18.01097	−18.46351
17	19427.08	20.83816	8.71E-14	−18.72050	−17.96189	−18.44228
18	19464.46	72.09299	8.53E-14	−18.74140	−17.93881	−18.44705
19	19478.84	27.67917	8.54E-14	−18.73982	−17.89325	−18.42934
20	19501.78	44.06581	8.48E-14	−18.74661	−17.85606	−18.41999
21	19529.51	53.16362	8.39E-14	−18.75808	−17.82355	−18.41533
22	19559.06	56.53184	8.28E-14	−18.77132	−17.79282	−18.41245
23	19568.19	17.43206	8.33E-14	−18.76461	−17.74213	−18.38961
24	19602.24	64.85385	8.19E-14	−18.78224	−17.71579	−18.39111
25	19620.02	33.80776	8.17E-14	−18.78399	−17.67355	−18.37673
26	19668.81	92.56869	7.92E-14	−18.81604	−17.66162	−18.39265
27	19686.00	32.55312	7.91E-14	−18.81720	−17.61881	−18.37768
28	19712.19	49.48835	7.83E-14*	−18.82716*	−17.58479	−18.37152
29	19726.18	26.37624	7.85E-14	−18.82520	−17.53885	−18.35342
30	19742.86	31.38774*	7.84E-14	−18.82586	−17.49554	−18.33796

*Lag order selected by the criterion.
LR, sequential modified LR test statistic (each test at 5% level); FPE, Final prediction error; AIC, Akaike information criterion; SC, Schwarz information criterion; HQ, Hannan–Quinn information criterion.

length of 2. However, considering the prevalence of AIC in the literature, a lag length of 28 is chosen for this research. Figure 5 demonstrates the impulse response function which is the response of one variable in the VAR model to a one-time shock to another variable, holding all other variables constant. The impulse response function is a useful tool for understanding the

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Table 4. Augmented Dickey–Fuller (ADF) test analysis

	***t*-Statistics**	Prob.*
Bitcoin return	−46.8151	0.0001
Gold return	−44.7092	0.0001
Oil return	−33.3750	0.0000
S&P 500 return	−14.4369	0.0000
Test critical values		
1% level	−3.4333	
5% level	−2.8627	
10% level	−2.5674	

*Significant at 1% level.

Table 5. Summary of response to Bitcoin return

	Crude Oil	Gold	S&P 500
Increase in Bitcoin	+	–	+
Effect size	0.60%	0.08%	0.20%

Table 6. Summary of the response to Crude Oil return

	Bitcoin	Gold	S&P 500
Increase in Crude Oil	–	+	–
Effect size	0.40%	0.08%	0.20%

Table 7. Summary of response to Gold return

	Bitcoin	Crude Oil	S&P 500
Increase in Gold	–	+	+
Effect size	0.30%	0.60%	0.10%

Table 8. Summary of response to S&P 500 return

	Bitcoin	Crude Oil	Gold
Increase in S&P 500	–	–	+/–
Effect size	0.30%	0.80%	0.08%

dynamic relationships between variables (Tsay, 2005). Table 4 presents the ADF test results. Also, summary of responses to Bitcoin Return, Crude Oil Return, Gold Return and S&P 500 Return are shown Tables 5–8 repectively.

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4.1. Response to Bitcoin Return

Figures 5, 6, and 7 present the impulse response to Bitcoin return.

The response of Crude Oil return to Bitcoin return is positive for most of the lags which means that an increase in Bitcoin return leads to an increase in Crude Oil return. In the first two lags, although the response of Crude Oil returns to Bitcoin returns decreases, the relationship is positive. The response is negative only at lag 10. The magnitude of the response is small, with the largest value being 0.006. This suggests that the relationship between Bitcoin return, and Crude Oil return is weak, and the impact of Bitcoin Return on Crude Oil return is limited. Generally, it can be said that Bitcoin returns positively affect Crude Oil returns, but it is weak and limited. A similar result was found by Liu and Naktnasukanjn (2022), finding that Crude Oil returns were positively affected by a rise in Bitcoin returns specifically in the early periods of COVID-19.

The response of Gold return to Bitcoin return is negative. It means that an increase in Bitcoin returns results in a reduction in Gold returns for most of the periods. For instance, an increase in Bitcoin returns causes to decrease in Gold returns for periods 3, 7, 8, 9, and 10 (a total of 5 periods out of 10) while it leads to an increase in Gold returns for the periods of 1, 2, 5, and 6 (only for 4 periods). The lag 4 is neutral. However, this response is not significant as the effect size is small, with a maximum value of 0.0008. Therefore, the connection between Bitcoin returns has a negative impact on Gold returns, but it is weak and limited. This result shows that Gold and Bitcoin can be used to diversify a portfolio as Gold returns are affected negatively by a rise in Bitcoin. Owusu *et al.* (2020) and Kyriazis (2020) found similar results by finding that gold and cryptocurrencies can act as hedging instruments.

The response of S&P 500 return to Bitcoin return is positive. An increase in Bitcoin returns results in to rise in S&P 500 for 9 periods excluding lag 9. After lag 1, the response volatility is between 0.000 and 0.001. The effect size is quite small after the first two lags. This implies that there is a low correlation between the returns of Bitcoin and S&P 500. Overall, Bitcoin returns have a positive impact on S&P 500 returns, but it is weak and limited. Similarly, Nguyen (2022) found that stock markets are highly affected by Bitcoin returns specifically in uncertain times and previous stock market returns. In a nutshell, although Bitcoin returns have a positive impact on the Crude Oil and S&P 500 returns, it affects the Gold returns negatively. The effect size is weak and limited.

4.2. Response to Crude Oil Return

Figures 8, 9, and 10 show the impulse response to Crude Oil return.

The response of Bitcoin return to Crude Oil return is negative which means that an increase in Crude Oil return causes a decrease in Bitcoin return. The result of the impulse response function shows that there is no relationship between the returns in the first lag. The magnitude of the response is small, with the largest value being 0.004. Although the relationship between Crude Oil and Bitcoin returns is negative, the effect size is very close to 0.000 for the lags of 5, 6, and 7. To put it simply, Crude Oil returns negatively affect Bitcoin returns, but it is weak and limited. This result was supported by Jareno *et al.* (2020) by found that Bitcoin returns are negatively sensitive to oil returns at low quantiles.

The response of Gold returns to Crude Oil returns is positive meaning that an increase in Crude Oil returns leads to a rise in Gold returns for most of the periods. For instance, the Gold returns increased for the lags of 1, 2, 5, 6, and 9 in response to Crude Oil returns while Gold returns declined for the lags of 3, 4, 7, and 10. The response is neutral at lag 8. The magnitude of the response is small, with the largest value being 0.0008. Therefore, there is a weak and limited relationship between the returns. As a result, Crude Oil return has a positive impact on Gold return, but it is weak and limited. A similar result was found by Tiwari and Sahadudheen (2015) by finding that an increase in Oil return has a positive effect on Gold return.

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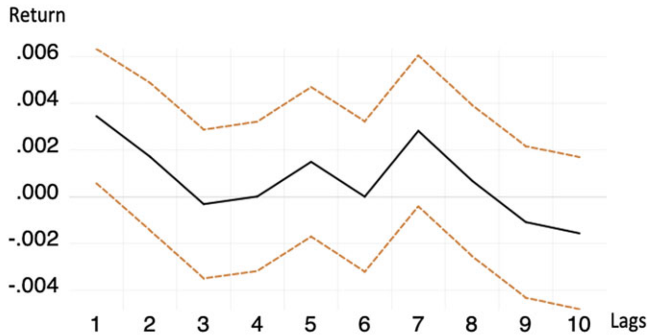

Figure 5. Response of Crude Oil return to Bitcoin return.

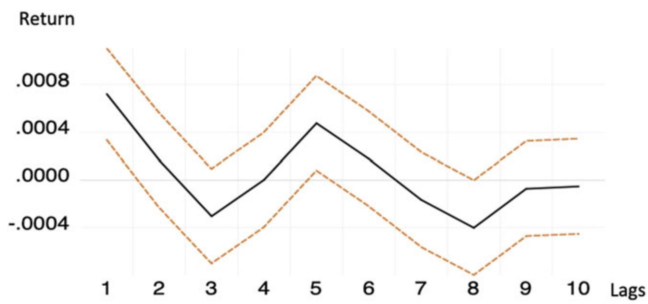

Figure 6. Response of Gold return to Bitcoin return.

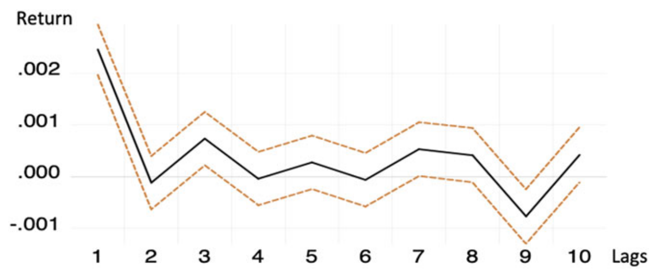

Figure 7. Response of S&P 500 return to Bitcoin return.

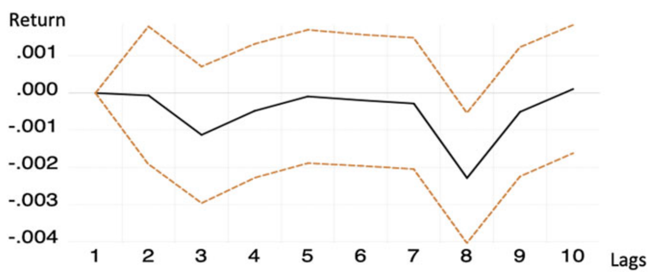

Figure 8. Response of Bitcoin return to Crude Oil return.

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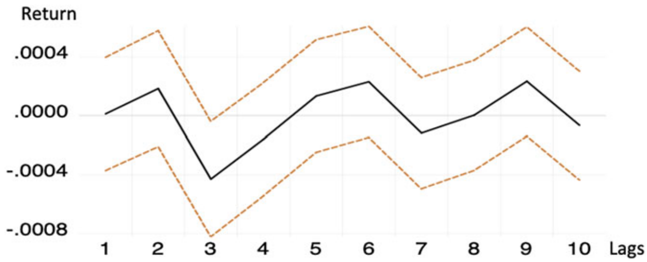

Figure 9. Response of Gold return to Crude Oil return.

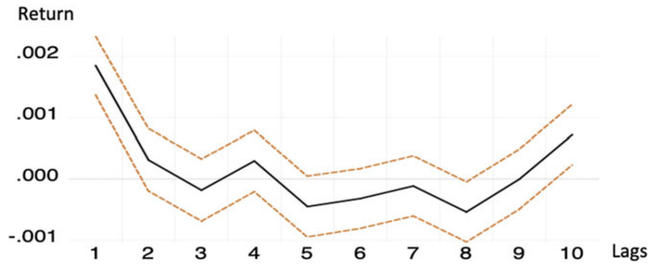

Figure 10. Response of S&P 500 return to Crude Oil return.

The response of S&P 500 returns to Crude Oil returns is negative, which means that for the majority of time periods, a rise in Crude Oil returns leads to a decline in S&P 500 returns. An increase in Crude Oil returns caused to decrease in S&P 500 returns at lags of 3, 5, 6, 7, and 8 while S&P 500 returns went up at lags 1, 2, 4, and 10 when Crude Oil returns increased. The response is neutral at lag 9. However, this response is not significant as the effect size is small, with a maximum value of 0.002. Therefore, although Crude Oil returns do have a positive impact on S&P 500 returns, this effect is not particularly strong, and the effect is limited. This result is supported by Ciner (2013) by finding that Crude Oil returns with less than 12 months of persistence have a negative impact on the stock market. To sum up, while Crude Oil returns have a positive impact on Gold returns, they have a negative impact on Bitcoin and S&P 500 returns. However, the effect size of these relationships is weak and limited.

4.3. Response to Gold Return

Figures 11, 12, and 13 present the impulse response to Gold Return.

The response of Crude Oil return to Gold return is positive, it indicates that an increase in Gold return leads to a rise in Crude Oil return. Specifically, in 6 out of 10 periods, an increase in Crude Oil returns corresponded to a rise in Gold returns, while in only 4 periods, a decrease in Crude Oil returns corresponded to an increase in Gold returns. However, the overall effect size was small, with a maximum value of 0.006, indicating that the positive impact of Gold returns on Crude Oil returns is weak and limited. According to Le and Chang (2012), the relationship between Oil and Gold returns is positive and statistically significant. In addition to this, there is a non-linear and asymmetric relationship between Oil and Gold returns (Kumar, 2017).

The response of Bitcoin return to Gold Return is negative, meaning that an increase in Gold returns corresponds to a decrease in Bitcoin returns. The impulse response function shows that there is no relationship between the returns in the first lag. The magnitude of the response is small, with the largest value being 0.003. In other words, the negative impact of Gold returns on Bitcoin returns is weak and limited. The relationship between Bitcoin and Gold returns is non-linear.

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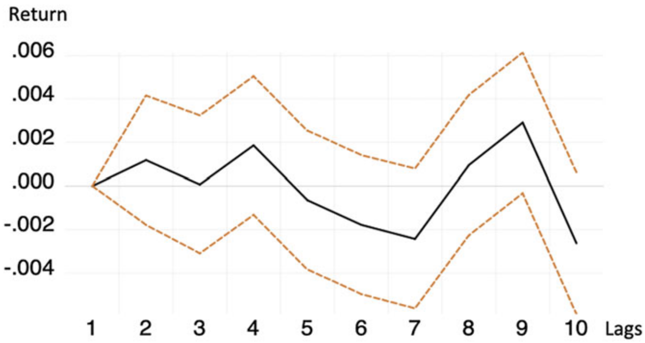

Figure 11. Response of Crude Oil return to Gold return.

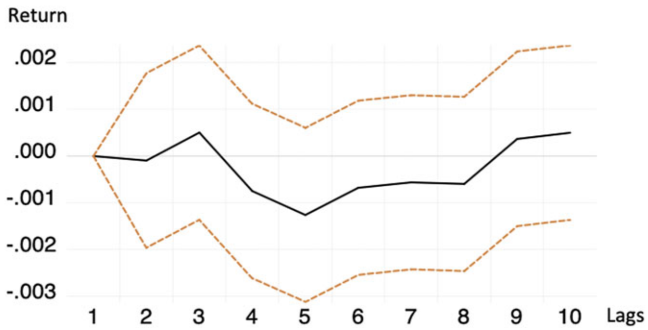

Figure 12. Response of Bitcoin return to Gold return.

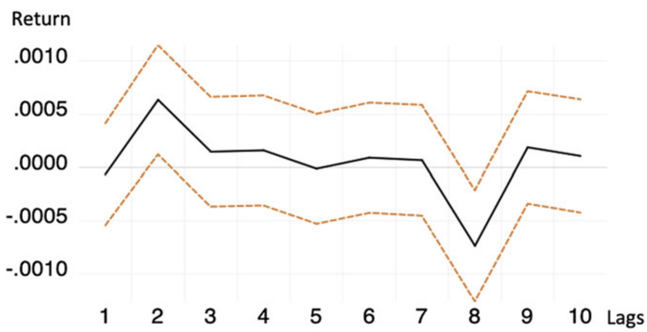

Figure 13. Response of S&P 500 return to Gold return.

A similar result was found by Zwick and Syed (2019), the return relationship is negative and limited because Bitcoin is perceived as a speculative asset.

The response of S&P 500 return to Gold Return is positive for most of the lags, indicating that an increase in Gold returns tends to correspond to an increase in S&P 500 returns. The only negative response is observed at lag 8. The effect size is small, with the largest value being 0.001, indicating a weak relationship between S&P 500 and Gold returns, and a limited impact of Gold returns on S&P 500 returns. According to Drake (2022), a rise in Gold return has a positive impact on S&P 500 return during periods of negative real interest rate. Therefore, S&P 500 and Gold can be seen as safe even instruments against negative macroeconomic variables. In conclusion, while

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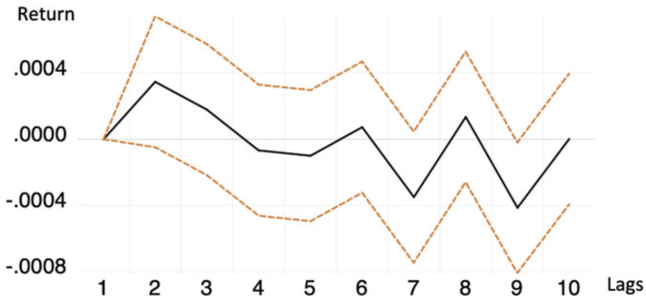

Figure 14. Response of Gold return to S&P 500 return.

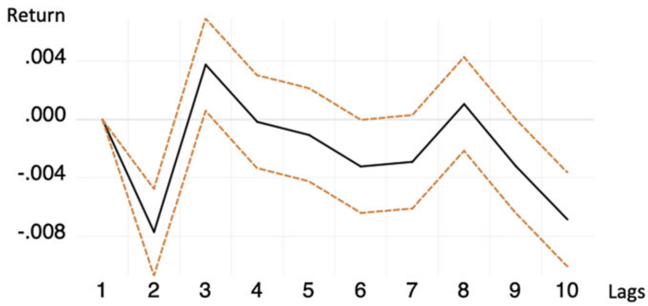

Figure 15. Response of Oil return to S&P 500 return.

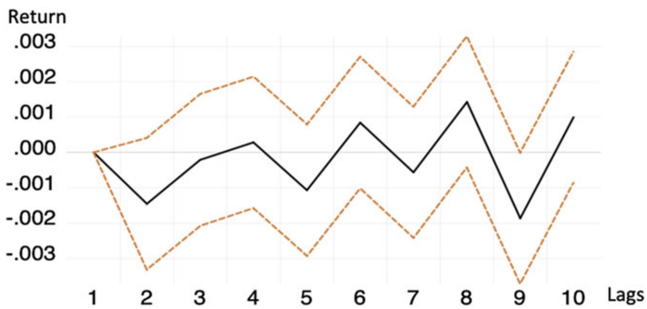

Figure 16. Response of Bitcoin return to S&P 500 return.

Gold returns positively impact Crude Oil and S&P 500 returns, they have a negative impact on Bitcoin returns. Nevertheless, the effect size of these correlations is weak and limited.

4.4. Response to S&P 500 Return

Figures 14, 15, and 16 show the impulse response to the S&P 500 return.

The response of Gold returns to S&P 500 returns is neutral. An increase in S&P 500 returns leads to an increase in Gold returns at lags of 1, 2, 3, 6, and 8 (5 out of 10 periods) while Gold returns decrease at lags of 4, 5, 7, 9, and 10 (5 out of 10 periods). The magnitude of the response is small, with the largest value being 0.0008. However, for the lags of 4, 5, and 6 the effect size is very close to 0.000. However, when stock return falls sharply, stock returns rise significantly (Miyazaki, 2019). According to Mamipour and Vaezi Jezeie (2015), stock returns have a positive effect on the

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Gold return in the short run (10 months) while the relationship between them is negative for the medium and long run.

The response of Crude Oil return to S&P 500 returns is negative, indicating that an increase in S&P 500 returns tends to correspond to a decrease in Crude Oil returns. The effect size is small, with the largest value being 0.008. In 7 out of 10 periods, an increase in S&P 500 returns corresponded to a decrease in Crude Oil returns, while in only 2 periods, an increase in S&P 500 returns corresponded to an increase in Crude Oil returns. In simple terms, while Crude Oil returns negatively impact Bitcoin returns, the effect is weak and limited. Like this research, an asymmetric and non-linear relationship was found between Crude Oil and stock market returns and there was a negative and limited relationship between Crude Oil and the US stock market (Sim & Zhou, 2015).

The response of Bitcoin returns to S&P 500 returns is negative, meaning that an increase in S&P 500 returns leads to a decrease in Bitcoin returns for most periods. Especially, an increase in S&P 500 returns causes a decrease in Bitcoin returns for periods 1, 2, 3, 5, 7, and 9 (6 out of 10 periods), while it results in an increase in Bitcoin returns for periods 4, 6, 8, and 10 (4 periods only). The response at lag 4 is neutral. However, the effect size is small, with a maximum value of 0.003, indicating that this correlation is not significant. Therefore, although there is a negative impact of S&P 500 returns on Bitcoin returns, the effect is weak and limited. According to Panagiotidis *et al.* (2018), the effects of stock return on Bitcoin return are mixed. In other terms, DJ, SSEC, and Nasdaq have a positive impact on Bitcoin returns while S&P 350, NIKKEI, and VXD have a negative impact on Bitcoin returns. To sum up, S&P 500 returns have a negative effect on Crude Oil and Bitcoin returns, but the impact on Gold returns is neutral. However, the magnitude of these correlations is weak and limited.

5. Concluding Remarks

This study used a VAR analysis of return data to reveal the volatility and dynamic interconnections among Bitcoin, crude oil, gold, and S&P 500.

Research Question 1*: Is there an interaction among Bitcoin, crude oil, gold, and S&P 500?*
Answer: Increases in the returns of crude oil, gold, and S&P 500 have a negative effect on Bitcoin's returns. On the other hand, a rise in the returns on Bitcoin and gold has a positive impact on crude oil returns while a rise in the returns on S&P 500 has a negative impact. The correlation between gold and S&P 500 returns is negligible, suggesting that there is no substantial influence on the returns of either asset. Gold returns are negatively influenced by an increase in Bitcoin returns, while positively affected by the increase in crude oil returns. Lastly, an increase in gold and Bitcoin returns positively influences the returns of S&P 500, whereas the returns of crude oil have a negative impact on S&P 500 returns. It is important to emphasise that these interactions among the financial assets are characterised by weak and limited effects.

Research Question 2*: Can portfolio risk be minimised through diversification by investing in Bitcoin, crude oil, gold, and the S&P 500?*

Answer: According to the findings from the VAR analysis, incorporating Bitcoin, crude oil, gold, and S&P 500 in a portfolio could potentially serve as a means to mitigate overall risk because the effect size of these relationships is limited and the observed relationships are weak. In particular, Bitcoin could be considered for inclusion with crude oil, gold, and the S&P 500 to help minimise portfolio risks. Additionally, including crude oil in combination with S&P 500 might contribute to reducing portfolio risks. Our findings highlight the interconnectedness of these asset classes within today's global financial markets and provide valuable insights for investors and policymakers. The results indicate bidirectional relationships among four different types of assets, suggesting that shocks in one asset's returns can impact and be influenced by the movements in other assets. Moreover, the analysis reveals dynamic responses and transmission mechanisms that operate over different time horizons. Understanding these interdependencies is crucial for constructing robust portfolio diversification strategies, implementing effective risk management techniques, and identifying potential hedging opportunities. However, it is significant to emphasise that the effect sizes of these relationships are relatively weak and limited, implying that caution should be exercised when applying these findings in practice. To enhance our understanding of asset pricing and volatility modelling, further research is necessary to explore additional factors and refine the analysis. Future studies could consider incorporating more assets, expanding the time period, or examining alternative modelling approaches. This would contribute to a more comprehensive understanding of the complex dynamics within the global financial system.

A potential limitation of the study lies in the approach to lag selection for the VAR model. While the aim was to capture a broad range of potential lagged influences by selecting a maximum lag of 28, this approach may lead to a model structure that is overly complex and challenging to interpret and validate. For future research, it is essential to acknowledge the importance of adopting a more parsimonious approach, which involves identifying lag orders that can be excluded based on both statistical criteria and theoretical considerations. By refining the lag selection process in future analyses, researchers can aim to strike a better balance between model adequacy and complexity, ultimately enhancing the interpretability and robustness of the findings.

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