Pupils' Use of Visual Mathematical Problem Solving Strategies

Elizabeth Loughran and Niamh McGrogan reflecting on ways to encourage mathematical thinking and visualisation in Year 1 pupils.

Elizabeth Carruthers and her team at Redcliffe Early Years Centre have researched the importance of children's free graphics (Carruthers and Worthington, 2011). Their findings led them to become strong advocates of supporting children to work mathematically using their own graphics to represent their ideas and explain their thinking. We were keen to explore how the Y1 children used visualisation to understand and work through activities involving telling the time to the hour. The Year 1 class had been exposed to some formal mathematical methods, but were encouraged to explore and present problems in their own ways during carpet discussions. The aim was to find out whether the children would choose to present their ideas visually if provided with the opportunity and resources with which to do so.

The class were divided into three core groups of 5 or 6 children working alongside an adult: the trainee teacher, the teaching assistant, or a university tutor. The structure of the groups was designed to ensure a mix of friendship groups, genders, mathematical confidence, and knowledge of spoken English. Each group was provided with an investigation relating to hour times, an area of the curriculum that the class had considered on the previous day, ensuring that the children had 'a foundation of subject-specific competences' in order to access the learning (Carden and Cline, 2015). Each group had access to pencils, coloured pencils, analogue clocks and plain paper. During the introduction, the children were informed that there was no single correct way to respond to the questions, they could use any of the resources, they could draw, discuss, write or work in any way to help them and their team. They were advised to share and discuss their ideas with the others on their table, ask each other questions and respond to each other without asking permission.

The hour-long lesson was divided into three twenty minute sections. Initially, the groups were asked to work through their problem together: the adult was an active participant in supporting their group to explore the problem, initially helping the children to read the scenario and then asking questions where necessary to prompt children to pursue their own and each other's lines of enquiry. During the following two sections the groups were mixed, with two children from the initial group remaining to introduce their investigation to the new children from the other two groups and to assume a facilitator role. The adult's role during the second and third sessions was to become a known and supportive observer, supporting facilitators in prompting their teams for contributions, and making notes.

Lemme the Ladybird

It is 2 o'clock. Lemme the ladybird is sat on the hour hand of a big clock. Which number is nearest to Lemme on the clock?

Lemme falls asleep and when he wakes up he sat next to a 6. The long hand is on 12. What time is it?

Lemme stays on the same hand and sings a song. He sees that the big hand is on 12, but Lemme is now on one more than 6. What time is it?

Lemme flies across to the big hand. Someone nearby says "it's 8 o'clock". Which number is Lemme sat next to? Which number is the other hand pointing at?

This investigation is based around a ladybird that travels around on the hand of an analogue clock. At various points he wakes up and looks at the other hand to establish the number nearest to him.

Initially the children discussed the question and drew ladybirds and analogue clocks on the large sheet of paper. For example, one child drew a ladybird with a rounded arrow, which represented the ladybird turning to look around. Many of the children drew circular clocks, numbered from 1 to 12, with 2 hands similar to the clocks used during the previous lesson. Four of the children drew a representation of the ladybird on one of the hands showing they were visualising the written problem.

In the second session, the facilitators from the core group instructed all of the children to pick up a clock and 'make two o'clock'. Fewer images were drawn during this session, perhaps because the plastic clocks replaced their need to draw their own representation. The facilitators in the final session were observed to be 'very quiet' and the children mostly worked independently using clocks to find their answers. They occasionally looked at the work of those next to them, without speaking.

The first of Pennant's (2014) four stages of problem solving is gaining an understanding of what is being asked, which could include drawing or acting out the problem. This seems to have been evident in this group, as they discussed the meaning of the question and drew images of key aspects. Saundry and Nicol (2006) noted that some children 'processed the mathematics through the act of drawing the "story" or problem situation', which may have been the intention of the child who drew the ladybird with the rounded arrow. The arrow seems to suggest an awareness that the task is to establish what the ladybird would be able to see when it moved in a cyclical motion. This could be an interesting example of a child trying to work out what can be perceived from a spatial position other than their own. There was a lack of discussion of the children's intentions at the time, so their specific reasoning was unclear, something we need to consider in future. Some of the children may have drawn pictures to represent their solution, whilst others may have been representing the problem, or using it to find a solution. Further exploration of the children's views of the purpose of their pictorial representations should provide us with further insights into their problem solving processes.

Buses

Buses are driven between 1 o'clock (lunchtime) and 12 o'clock (midnight).

They don't leave every hour, but a bus leaves every <u>two hours</u> between 1 o'clock and 12 o'clock.

If the first bus leaves at 1 o'clock. What time are the other buses?

How many buses are there in a day?

This group explored the frequency of buses that could travel during a given period if they left on the hour every two hours. The investigation was developed to encourage the children to think about intervals of time, which is an important, but sometimes difficult, concept for children to understand. The investigation was designed to encourage the children to combine their knowledge of time with their ability to count in twos, merging different aspects of their existing mathematical knowledge and potentially prompting some children to record their thinking to help them understand how the two aspects could work together.

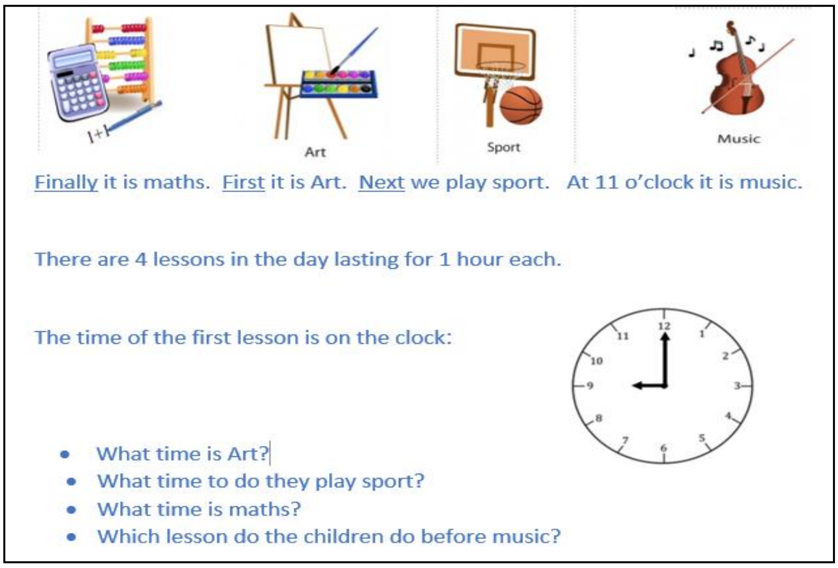
The question provided the children with some specific information, such as "the first bus leaves at 1 o'clock", which they all read and seemed to understand since they used this time to work out the times of the subsequent buses. However, they then appeared to forget about the initial time, perhaps because it was given to them rather than being worked out, and they counted one fewer buses in a day (5 buses).

A facilitator in the second group, advised the children to 'go up in twos', sharing her tested method with the group. Her instruction prompted the children to start counting in twos and they reached a shared solution, which was the six buses we had been expecting. Several children wrote '6 buses', next to their jottings. However, it would seem that the facilitators then read their original solution to the group as well since several children wrote their own finding '6 buses' and then wrote 'There are 5 buses', suggesting they were then recording their facilitator's phrase.

The children working on this investigation were largely focused on the final solution. This made the investigation interesting because the core group's oversight meant they miscalculated the number of buses. As such, the child facilitators who led the subsequent two sessions were equipped with a mistaken understanding of the solution. Interestingly, the children in session two did not question the difference between the answer they had found when following the method and the answer their facilitator had given them. This would indicate the need for children to acquire skills in questioning mathematically and to search, and insist upon, evidence to support outcomes. This would support the development of mathematical inquiry and a greater importance being placed on process rather than outcome. The facilitator in the second group did indicate some significance being given to the route to the answer, since she advised her group to 'count in twos', which enabled them to find and record their own solutions.

The lack of drawing of the problem solving process by the children may have been because the children were given the solution and therefore did not feel they needed to draw to help work it out. Saundry and Nicols (2006) point to a limitation in the teaching of primary maths that can lead to a focus on the solution as the end-product rather than the emphasis being placed on understanding methods and the problem solving process. The significance to teachers is clear – an emphasis on outcomes rather than processes can lead to misconceptions and reduced mathematical questioning and reasoning.

A School Timetable



This investigation was based on a school timetable, where the children matched the times with the subject and answered questions relating to the order of the lessons. The group had used relevant skills and pieces of knowledge in class, but as with the Buses investigation this activity combined existing knowledge and was intended to prompt children to record ideas to combine their knowledge. The group largely recorded their responses on individual sheets of paper. Most started by recording key times or sentences copied from the question, which may reflect an attempt to understand the nature of the question and understand the 'story' they were given.

All the children then drew two columns. It could not be determined which child chose this layout first, but it was clear that children on the table were watching each other and all copied. The information that the children chose to record in their two columns varied, some put times first, others put subjects first, and others did not have a clear distinction between the content of their columns. This group included two children with limited spoken English who focused on visual cues: one pointed to the image of the violin and the other made the time drawn on the sheet on her clock.

The facilitators in the subsequent sessions used different approaches when addressing their peers. One child made 9 o'clock on his clock, showed it to everyone and said 'art' to convey the answer. The second facilitator in the second session focused on the layout of the children's work. He explained that 'you need to do some boxes and the times and the answers'. The second child looked at one child's answer and said 'right', then at another and said to the observer: 'she needs to swap arms'.

In the third session, one of the facilitators handed out the clocks, whilst the other provided instructions: 'draw 2 squares, then show me your paper'. One of the 'students' in this group, repeated the statement 'there are 4 lessons in a day', perhaps trying to make sense of the question before deciding upon his preferred approach, but he received no response. It was interesting that several of the children recorded digital times and two recorded 'pm' in their columns, since neither of these conventions had been taught in school. This suggests the children had associated the time problems with prior knowledge taken from outside of the classroom, perhaps learnt from discussions with their friends and families or exposure to digital technology. This supports the view of Carden and Cline (2015) that children's prior knowledge is essential when they are problem solving, but reinforces the significance of constructivist arguments about individuals' schema or webs of ideas that are built upon when learning takes place. The responses of those children for whom English is an additional language may suggest an enhanced significance of images where language is less clear.

The first facilitators had associated the visual portrayal of the time on the clock, which reinforced the structure of the teacher-led lesson on the previous day, when the children were given a time and asked to create it using their clocks before displaying it and reading the time depicted. In contrast, the second facilitator's focus on the way to record solutions suggests a solution-focused approach, which contrasts to the shared understanding of the method. The second child may also reflect an emphasis in schools on the neat layout of work rather than the mathematical thinking it reveals. A discussion designed to elicit the mathematical reasoning of the children may have had an impact in this investigation, requiring the children to explore their understanding of their processes and justifying these mathematically.

Concluding reflection

This study considers one lesson with one class, so further study would be needed to find out to what extent the mathematical strategies the children used are representative of their personal preferences and those of other children. The case studies carried out by Saundry and Nicols (2006) involved filming the responses of the children. By having access to video footage it may have been easier to follow the way in which the children interacted, including looking at who they observed prior to drawing their own images and thus allow for a deeper understanding of the children's processes.

Yet despite the limitations imposed by the research method, there were some themes beginning to emerge, such as the significance of previous schema to children's ability to work through mathematical investigations and the significance of the combination of language and visuals to enabling these children to access, work through and share their thinking around the problem with others. It was particularly interesting that the children reverted to mirrored methods when acting as facilitators, such as using plastic clocks or tables. This would indicate that teachers should attempt to give children the opportunity to engage with problems in a range of ways to ensure their thinking is not limited to any one method, but that all mathematical reasoning is valid. The children's visualisation strategies varied depending upon whether they were working through a problem for their own purpose, or sharing thinking with others as a more experienced 'facilitator'. This perceived purpose of the activity was significant in shaping the way they worked. For the initial core groups, the need to work mathematically was important since they were working through the investigation to enable them to teach during the subsequent sessions. All observers identified their initial core group as the most engaged in the problem. In subsequent sessions, the need for an outcome seems to have become more important. This is most clearly apparent in the final group working on Investigation 2 who wrote down an incorrect solution because it was dictated to them by their facilitator.

Carruthers (2011) suggests there can be a divide between children's free graphics and the methods they are taught in schools. There is some evidence for this from the lesson, since the children varied their approaches depending on whether they were 'working mathematically' with their core team or 'being a facilitator'. Perhaps the children did not see their initial visualisation and approaches as worthy of sharing with their peers, since their methods did not necessarily fit with the methods they had observed in previous teacher-led lessons. This led to problems, particularly in group 2 where the children used the core group's answers incorrectly.

A notable outcome is the need for this open approach to become an established classroom practice, to support children in establishing their own reasoning strategies. Creating more opportunities for children to reason mathematically using their own methods may inspire a greater confidence in the application of knowledge to new contexts, thus enhancing children's mathematical understanding and awareness.

References

Carden, J. and Cline, T. (2015) Problem solving in mathematics: the significance of visualisation and related working memory, *Educational Psychology in Practice,* 31(3), 235-246.

Carruthers, E. and Worthington, M. (2011) Pedagogy that supports Mathematical play. *In*: Carruthers, E. and Worthington, M. Developing *children's mathematical graphics: supporting early mathematical thinking*, Maidenhead: Open University Press, 155-173.

Pennant, J. (2014) 'Developing Excellence in Problem Solving with Young Learners' *NRICH: Enriching Mathematics* [Online] Available from: https://nrich.maths.org/10865 [Accessed 5 August 2016].

Saundry and Nicol (2006) Drawing as problem-solving: Young children's mathematical reasoning through pictures [Online]. Available from: http://www.emis.de/proceedings/PME30/5/57.pdf [Accessed 5 August 2016].