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Chapter 11: **'This is the safe. It has a number and no one else knows it' – Playing with mathematics**

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Vignette

The research setting in which this vignette was documented, is a maintained Nursery School in an inner-city district of a large multicultural city in the southwest of England[i]. The nursery school welcomes families from many different ethnic backgrounds, & and at the time of data collection children attending the nursery spoke thirteen different languages. The children in this observation are aged three to four-years of age and in their final year of nursery school. Both boys' first language is English.

The vignette focuses on an episode of social pretend play in which the boys' collaborative dialogue grew from Isaac's personal interest in security safes, one of his many *funds of cultural knowledge* (Moll, Amanti, Neff & Gonzales, 1992; Worthington, 2018; Worthington & van Oers 2016). During the course of their play the boys made reference to many aspects of number and quantity, culminating in communicating ideas through emergent mathematical inscriptions (Carruthers & Worthington, 2006).

Throughout the year Isaac initiated and engaged in pretend-play more than any other child in the study, his play often highly complex and sustained. Isaac's interest in security technologies such as padlocks, security cameras and safes arose from his first-hand experience at home and through his father's building work. He also has well-developed knowledge of money though his father's new work with his micro-brewery, involving deliveries, invoices, payments and counting cash, all activities in which Isaac has been involved. Shortly before this observation – and knowing of Isaac's interest in safes - his teacher Emma had brought in a small safe for the children to explore[ii].

The vignette is drawn from qualitative data gathered for case studies for doctoral research, investigating children's beginnings and development of their mathematical meaning making and their emergent understandings. This research underscores the potential of pretend play for *cultural-conceptual* mathematical learning and is founded on a Vygotskian cultural-historical and social-semiotic perspective (Vygotsky, 1978; Worthington & van Oers, 2016). Longitudinal, ethnographic data were gathered during the course of one year. These data comprise written observations of children's behaviours, talk and learning and children's inscriptions, all the children's explorations relating to their personal cultural knowledge.

The following vignette is taken from Worthington and van Oers (2016, pp. 58-59).

Jayden and Isaac decided to transport wooden blocks on a trolley. They moved a small cupboard that stood nearby to create a 'safe', and then placed a keyboard and clipboard on top of it.

When another child came and took away one of the blocks, Jayden noted this down on his clipboard by writing wavy lines. He took the piece of paper off the clipboard and placed it in their safe, then tapped several keys on the keyboard, repeating this process every time a child removed one of the blocks.

Isaac announced, “This is the safe. There’s a key, only one - and you press it here and it opens. It has a number and no one else knows it - one, one, eight, seven, zero, six. It’s rather difficult to remember.”

Jayden brought over some coins and cheques to put in their ‘safe’. Isaac found a calculator and stuck it onto their safe with tape, explaining, “You need to press the buttons to get in the safe… it’s four, nine, seven, nine.”

Jayden pressed some numbers, making a “beep, beep, beep” noise as he opened the safe. Closing the doors Jayden asked, “What’s the closing number?” Then said, “one, nine, five, two” as he pressed buttons on the calculator.

Later Jayden told his teacher Emma, “You need to give me one, nine, five, two.” Emma explained that she didn’t have that amount in cash but could write a cheque and Isaac replied, “I need hundreds of pounds in cash!”

Emma managed to find a selection of coins in her purse and gave them to Jayden who said, “Okay! We need to fill the box for one, five, six, zero pounds.”

“It’s a skip!” Isaac said excitedly as the box filled, “It’s getting too full! It’s more than one, five, six, zero pounds!” Isaac asked his teacher to write ‘one, five, six, zero pounds’ on the box, and looking at the numbers and symbols he took a great interest in what she had written.

After several days creating and playing with their safe, Isaac used it to store “important information”. He decided to write down the number of blocks being removed from the block area, counting, “One, two, three, gone! Gotta write it down and put it in the safe.”[iii]

Three questions to assist readers in reflecting on the vignette

1. How can teachers ensure that they understand and value children's "funds of cultural knowledge" in relation to mathematics?
2. How might we support open opportunities for rich mathematical learning though play, particularly in settings where there are 'downwards pressures' from school, for more formal approaches to mathematics?
3. Provide an example of another pretend play situation you have observed, and discuss how professional practice can support meaningful mathematics learning embedded in that play.

Commentary 1: Pretend play as a context for mathematics

Author: Maulfry D. Worthington

The head teacher of this nursery school is committed to ensuring a democratic and open ethos, staff valuing and supporting children's self-initiated ideas, choices and decisions and how they express them. Mathematics and graphicacy (i.e. children's own drawings, maps, writing and mathematical inscriptions) have a very high profile and are valued and sensitively supported by staff[iv]. Especially noticeable is the staff members' views of the child as "a *rich child,* active, competent and eager to engage with the world" (Dahlberg, Moss & Pence, 2007, p. 7), sharing a belief that how we construct our views of the young child and of early childhood matters. Rather than narrow curriculum goals, with knowledge and skills transmitted *from* adults *to* children, learning is understood as social, cultural and collaborative. Teachers encourage children to initiate ideas, valuing and supporting their complex thinking through rich dialogue and emphasising the intellectual over academic, and children's meaning-making over 'school-readiness: this philosophy is the headteacher's.

Free and spontaneous play is an essential feature of this nursery school. Pretend play is neither themed nor planned by adults and is a popular choice for many of the children, their impromptu play frequent and often complex. Hewes (2014, p. 280) writes that for children, "adaptability, control, flexibility, resilience and balance result from the experience of uncertainty, unpredictability, novelty and non-productivity. These [are] essential dimensions of young children's spontaneous free play". Such play also allows children to be responsive to diverse aspects of their cultural knowledge within their play narratives, including their mathematical knowledge (Worthington & van Oers, 2016). Others who have investigated young children's social pretend play and the mathematics arising within it include Carruthers and Worthington, (2006; 2011); Cook, (2006); Munn and Kleinberg, (2003); Munn and Schaffer, (1993); van Oers, (2010; 2013) and Worthington, (2018; 2020).

In this nursery school children self-initiate their mathematical ideas through their pretend play indoors and out, also instigating their own ideas and choices in open, adult-led small groups. In contrast to some of the previously published research into pretend play and mathematics by other researchers (Gifford, 2005), the children often choose to explore and communicate mathematical ideas freely through talk and graphical inscriptions within their rich play narratives. The level of the children's achievement in mathematics at this nursery school (relating to national assessment procedures and government inspections) has been repeatedly found to exceed expected outcomes for children of this age.

Danniels and Pyle (2018, p. 1) point out that "play-based learning is distinct from the broader concept of play". They suggest firstly, that the term *play-based* encompasses children's own free play, initiated and developed by the children themselves, "voluntary, internally motivated, and pleasurable". However, rather than the common use of the term

"fun" when referring to children's play, our observations of young children's spontaneous and free pretend play is that it can be both enjoyable, and, at the same time also deeply serious and engaging for them.

A second interpretation of *play-based* learning identified by Danniels and Plye is of *guided play*, its most distinctive feature "who has control over the play activity" (p. 1). In England the term *play-based* is used only in teacher planned (and often teacher-led) activities with specific learning goals in mind, and often provides very little opportunity for children to freely make choices in that 'play'. In this vignette Isaac and Jayden had complete ownership of their play in respect of the location in which they chose to play, the friend with whom they played, the resources and materials they chose to use, the focus and development of their play and its duration. Our argument is that it is only children's truly spontaneous free play (within an open and supportive learning environment), that embodies important features of play. Such play allows children to explore their existing cultural knowledge, and can lead to such rich learning.

The statutory English *Early years foundation stage curriculum* (EYFS), (DfE, 2017) covers the period from birth to five years but in reality, almost all four-year-old children are in a "reception class" in a primary school[v]. One of the mandatory areas of the curriculum is mathematics, delineating "counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measure" (p. 8). The curriculum advises "play is essential for children's development, building their confidence as they learn to explore, to think about problems, and relate to others. Children learn by leading their own play, and by taking part in play which is guided by adults" (p. 9). However, although play is officially acknowledged, it may be fleeting and seldom includes children's mathematical explorations (Gifford, 2005).

The narrowing of the curriculum requirements and the "school readiness" agenda have resulted in a reduction of rich and sustained pretend play in most settings, and play is increasingly marginalised[vi]. In common with many countries, England appears to share in an escalation of prescribed "skills-based" teaching, often resulting in restricted understandings of children's own mathematics and only limited opportunities for rich play.The increasing politicisation of education risks losing sight of young children's needs and development (Worthington, 2020). Citing Moss (2013, p. 9), Robert-Holmes (2015) concludes "complex holistic child-centred principles, sensitive pedagogies and assessments [are] in danger of being marginalised [and have] the potential to reduce the rich competent child (and teacher) to a "measurable teaching subject" (Ball and Olmedo, 2013, p. 92)" (p. 11). One of the most worrying outcomes of the pressures of "schoolification" of early years education, is that taught mathematical "skills" are often unrelated to children's development and existing cultural knowledge.

Culture and mathematics

Locating two ways of appropriating cultural knowledge, Vygotsky (1978) highlighted how children do so directly through their experiences in cultural and social situations and practices (leading to spontaneous, "everyday" concepts), and through instruction (leading to schooled, "scientific" concepts). For Vygotsky, pretend play helps in "bridging" these concepts (Worthington & van Oers, 2016, p. 53). It offers potentially rich social contexts that "situate" learning, allowing children to explore and draw on their existing cultural knowledge of mathematics (Worthington, 2018). Bishop (1988b) argued that in the past, "the conventional wisdom was that mathematics was "culture-free" knowledge" (p. 146). Identifying playing as one of his "universal activities of mathematics", Bishop proposed that,

> This may seem initially to be a rather curious activity to include in a collection of cultural activities relevant to the development of mathematical ideas… *It is even more important to include it, when considering mathematics from a cultural perspective. One is forced then to realise just how significant 'play' has been in the development of culture*. (p. 149-150, emphasis added).

Mathematics in pretend play

In England the term "play" is commonly used to refer to a variety of child-initiated activities such as puzzles, play with sand and water and blocks and bricks, and, whilst they have value for mathematics, for Vygotsky *pretend play* is the "leading activity" for young children, its influence on their development "enormous" (1978, p. 96). Play scholars acknowledge play as led by and belonging to children (van Oers, 2013). However, Gifford (2005) found many researchers in England identified a striking lack of mathematics in pretend play. Paradoxically Worthington and van Oers (2016) found that play that is *owned by* children (rather than planned by adults, and without adult goals for mathematics), is frequently rich in contextually meaningful mathematics through their talk, behaviours and graphical representations, their play narratives arising freely and spontaneously and shaped and developed by them. In this instance the mathematical concepts the boys explored included *number* and *quantity, zero, money and capacity*, all explored in a relevant context that grew from and were related to their own experiences and interests.

In their analysis of three to four-year old children's pretend play Worthington and van Oers (2016) identified connections between the home cultural knowledge they brought to their play narratives and the mathematics they explored in their play. Their findings revealed also the extent of the children's spontaneous mathematical explorations within their pretence, as the vignette in this chapter shows. Underpinned by Vygotsky's (1978) cultural-historical (or socio-cultural) theory, the research on which this chapter is based regards concepts developing in tandem through interactions with cultural systems.

Playing in mathematics

Identifying six "universal" activities of mathematics, Bishop (1988a) specifies *counting, locating, measuring, designing, playing* and *explaining*. In Bishop's terms, the mathematics the boys explored in this vignette included *counting*, *locating*, *designing*, *playing* and

*explainin*g. Bishop defines *playing* as "games; fun; puzzles; paradoxes; modelling; *imagined reality*; rule-bound activity; hypothetical reasoning; procedures; plans, strategies; cooperative games; competitive games; solitaire games; chance [and] prediction" (Bishop, 1991, p. 101, emphasis added), writing that, "once the play-form itself becomes the focus… then the *rules*, procedures, tasks and criteria… are the products of playing" (p. 45, emphasis added). However, rather than a feature of mathematical activity, play scholars regard "playing" as *a behaviour, an activity and a context*. Children's spontaneous and free pretend play has the potential to embrace children's mathematical thinking: Seo and Ginsburg (2004, p. 25) emphaisise that adults need to "take the child's perspective, understand the child's current intellectual activities and build on the mathematics to foster the child's learning". However, unlike Bishop's perspective, whilst children may be engaged in pretend play, there is no guarantee of meaningful mathematical learning (for example, see Gifford, 2005), but the play depends on the culture and philosophy of the setting, and the teacher's intentionality (Hewes, 2014). For additional means of consolidating and enriching the children's mathematical understandings, see commentary 2.

Vygotsky (1978, p. 190) showed how children's pretend play, "depends on rules … it is the sense that children make of a particular cultural situation and its rules that motivates their actions". Vygotsky's socio-cultural theory is exemplified well in this vignette. The play and learning are fully social, allowing exchanges of ideas through dialogue, actions and sharing of a inscriptions: it is cultural in that its freedom permits the children to readily explore and extend their funds of mathematical knowledge. Bishop's (1988a) "'imagined reality'" also encompasses *rule-bound activity*, in which according to van Oers (2013, p. 191), the rules relate to "how [children] interact, how they use their tools, or how to organise the play"[vii]. Following Vygotsky, van Oers (2013, p. 191) identified three important features of pretend play, its *rules, degrees of freedom* and children's *involvement,* features we use here to interpret aspects of the vignette. The rules of pretend play are shown below:

Social rules (how to interact with one another)	The boys' turn-taking soon establishes them as equal partners in contributing ideas and cultural knowledge.
Technical rules (how to use a tool or piece of technology)	Isaac introduces the idea that the safe is opened with a key, at the same time referring to a "*difficult to remember*" number to open it. Isaac understands that some safes have a digital keypad to open and secure them. He fixes a calculator to the door of their "safe", explaining that its numerical buttons are to be pressed "*to get into the safe*". Jayden makes a "*beep, beep, beep*" noise, perhaps associating their safe with other technologies that make this noise.

Conceptual rules (i.e., rules with a conceptual basis)	When another child removed a wooden block, Jayden showed he understood something of subtraction by "writing" down what had occurred, his wavy lines resembling adults' writing. The boys explore ideas of number strings as codes for opening and closing the lock. Isaac refers to "*zero*" three times, and Jayden once. Isaac understands that a long number with six digits will be "*rather difficult to remember*". Jayden knows that valuables such as money are kept in safes. Jayden understands that a large amount of money would be needed to fill their box to capacity. Isaac understands that "*important information*" is kept in a safe".
Strategic rules *(*rules that support the course of an activity	The boys know that they are free to use and move resources and furniture as they wish. Isaac is interested in how his teacher represents "*one, five, six, zero pounds*". The strategic rules are implicit in the course of this play episode.

The boys' conceptual rules of mathematics include aspects of number (*subtraction, number strings*, *zero*, *money*, *quantity, graphical representations of quantity,* and *capacity)*. Reflecting on her observation, their teacher wrote "… both boys were using their experiences and understanding of numbers in a real situation… Jayden, in awe of large numbers in relation to money, knew that £156.00 was a large amount of cash" (Worthington & van Oers, 2016, p. 59).

Degrees of freedom

Van Oers (2013) writes that "the degrees of freedom" within play allow children to make their own choices concerning their "actions, tools and rules, etc.," (p. 191). In contrast, many children's experiences are circumscribed by restrictions on play that reveals only, "*adults' perceptions* of children's interests, rather than children's authentic and immediate interests that have personal cultural meaning" (Worthington & van Oers, 2016, p. 52, emphasis in the original). Weisberg, Kittredge, Hirsh-Pasek, Golinkoff and Klahr (2015) refer to adult planned activities as, "*disguised as play* … [such as] chocolate-covered broccoli" (p. 10, emphasis added).

Levels of involvement

Deeply involved in their play, Isaac and Jayden continued to explore the same narrative on several subsequent days. Van Oers (2013) regards pretend play as an activity in which children are highly involved, and this level of involvement is evident in this vignette. In this nursery school teachers value children's pretend play and support their meaning making through reflecting on their written observations; they engage in collaborative dialogue with them, and frequently model mathematical signs and representations in meaningful contexts. This professional community of practice appears to create a learning "hot spot", children instigating pretend play and exploring meaningful mathematics embedded in that play[viii].

Commentary 2: The mathematics teacher as key player

Authors: Elizabeth Carruthers

I am the headteacher of the nursery school and National Teaching School in this vignette, therefore my commentary is from a leader's perspective. I am mainly focusing on analysing the pedagogy of this play episode[ix].

This vignette is situated within the vibrant research culture of the nursery school, which enables democratic practices that respect the professional identities of the teachers (Osgood, 2006) and includes children as participants in their learning (Pascal & Bertram, 2009). Pascal and Bertram emphasise the power of practitioner research as the researcher can uncover the many nuances of everyday lived experiences with children and their families. This vignette is from the same nursery as in chapter 2, but differs since it is about children's imaginary play. It is taken from an extract of a child's learning diary, where notable observations of children's experiences in the nursery are made. They form part of the assessment process of this nursery school. This is one of many mathematical play sessions that this extremely skilled teacher, has observed (Butcher, 2017; Carruthers & Butcher, 2013). She uses these observations not only to assess the children's knowledge but to plan from them.

The nursery school is committed to children's free and imaginary play (Carruthers & Butcher, 2013). In this play episode the children are leading, using their imagination in pretence. Here they use their own mathematical knowledge producing *their* mathematics*:* this is different from traditional school mathematics in England where the focus is on teaching a pre-determined curriculum (DfE, 2013). In this vignette the children make the rules and there are no right or wrong answers. They are wallowing in play ideas and, in this episode, are using their previous knowledge to try and understand the concept of keeping money in a safe. As Steffe (2004) writes "the mathematics of the children emerges from within children and it must be constructed by children" (p. 35).

The mathematics in which the children are engaged is not traditional number knowledge of learning numbers in sequence or of standard calculations, but within the concept of safes and monies. Their numbers are not in the traditional order; you need to know the code and the children make up their own codes. This aspect seems to have fascinated the children, and in this short vignette of spontaneous pretend play, the children have used seven different combinations of the numbers, one to nine. They did this with fluency and are already meeting the expected outcomes of the EYFS (DfE, 2017) which are, for example, that children should show an interest in numerals in the environment and representing numbers; use number names and language spontaneously; show curiousity about numbers by asking questions and offering comments.

The teacher

In conversation with Emma about her pedagogy she says, it is not about "teachery teaching" but it is about "being with the children". This resonates with teaching that is attached (Carruthers, 2017), where the children and their teacher are conceptually and contextually connected. There is also a calm element within Emma's description of her pedagogy, since she is not rushing to achieve a teaching objective. She has personal knowledge of the children, their family life and their personal histories, and this makes it easier to understand their actions and thinking (Moll et al., 1992). She responds to the play spontaneously; at the same time, she understands the richness of what is happening and make notes of the play action and conversation. She is an *insider* in this scenario and a participant observer (Carruthers & Worthington, 2011). Without the written observations made at the time, reflection on this action would be difficult (Schon, 1987). Emma's acute understanding of what was important to note is vital to support the children in their continuous learning about safes and monies. For staff professional development it is also crucial to discuss this vignette, to add to their knowledge and to our knowledge through this chapter. This observation is a nugget of enlightenment for us to dissect, reflect, rethink and to learn from and it is important for the teacher's professional identity that we acknowledge this contribution (Pascal & Bertram, 2015).

The teacher as researcher

This teacher has been researching children's play experiences and the link with imaginary play and mathematics (Carruthers & Butcher, 2013). The nursery school encourages and enables teachers and practitioners to pursue research interests and provide space and time for them to participate in professional conversations in what Wenger (1998) might term, *a community of practice*. In a conversation with this teacher she said that,

> Play is like magic and when children get together the magic begins again. They can be anywhere at the park, on the boat. They just collaborate, and even if they have never met each other, they know they can join in, new ideas are thrown into the pot and are taken up, or not.

It is this appreciation of children's play that this teacher demonstrates through her observations, enjoying this thoughtful episode of child interaction. This is part of the pedagogy that makes play solid and useful to children (Carruthers & Worthington, 2011).

It is also about the teacher wanting to know and creating her own understanding, whilst deepening her knowledge about play. She not only uses her "on the ground" awareness of children's play, but is also aware of other play perspectives through reading relevant literature and discussing with colleagues; the combination is powerful (Carruthers, 2015). It creates new knowledge and understandings and continues to make play interesting to the teacher who can articulate her knowledge to others. For many children in English schools, play can be so structured that it becomes far removed from children's own play (Rogers, 2010): it is not encouraged or facilitated in many schools after the age of five as the children are in more formal education (Ofsted, 2017). There is a dearth of understanding about play

(Rogers, 2010). Added to this, teachers and practitioners are not confident in mathematics and are unaware of the potential of the mathematics in play and therefore may not respond to it (Pound, 2008; Markovits and Forgasz, 2017). It is unusual to find documentation of children's spontaneous pretend play sessions that include spontaneous mathematical play: however, Worthington and van Oers' research found that in a high proportion of their play observations in this nursery setting, children were engaging in a variety of aspects of mathematics.

Resources

Previous to this imaginary play episode, Emma invited mathematical happenings through resources by bringing in a safe, which she thought Isaac might be interested in. The children had already discussed and asked questions about the idea of a safe before this vignette occurred. In this present vignette the children became more interested in the concept of a safe and focused on a critical aspect, the number code to open the safe. Isaac knows the issues around number codes as he says, "it is rather difficult to remember" this secret number.

A key feature of the pedagogy of play that this teacher shows is "her receptivity to the unpredictable" (Fochi, 2019, p. 342). Emma accommodates the pretence and adapts to the situation as she offers alternative solutions i.e., a cheque which Isaac appears not to accept by quickly saying, "*I need hundreds of pounds in cash!*" Emma quickly responds by finding money in her purse, eager to keep the play going. Cheques have previously been a resource that the children had found useful in their play, but the coins seemed to suffice instead of notes, and are accepted by the children as part of the imaginary play. This highlights the importance of pretence in accommodating substitutes instead of real things. This adds to the children's growing knowledge of finance and monies. The resources are therefore never random, but authentic aspects of everyday mathematics of the world and acknowledge the children's threads of thinking at the time.

Availability of graphic materials

The resources of accessible graphic materials enable the children to use these tools in their play, and to communicate as they see adults communicate, in their world. It is part of their enculturation to the symbolic systems of their world (Kress, 1997). In this nursery this graphic equipment is always available in various forms. The walls of the nursery are covered with children's own free graphical explorations, rather than the finished intricate drawings for which the Reggio Emilia nurseries in Italy are renowned; it is the emerging and experimental graphics that are displayed. Children investigate the forms of lines and other early marks (Matthews, 2003) and here they also explore their own invented and standard signs and symbols (Carruthers & Worthington, 2005).

Modelling

A pedagogical strategy that this teacher employs is naturally modelling, in authentic situations, mathematical signs in play context. For example, she wrote down the numbers

Isaac requested "*and looking at the numbers and symbols he took a great interest in what she had written".* Humans, including children, possess an innate ability to learn by looking at and observing others (Meltzolf & Williamson, 2010). The teacher was modelling how to write numbers and their appearance in the context of an authentic purpose.

No limits on the mathematic learning

Emma does not limit the children's learning but encourages them to go far as they want within the concept in which they are currently involved. She is raising expectations of mathematical knowledge, in a broad sense, as the children develop the play by widening the scope of what is in the safe to "important information". This is in sharp contrast to what Jordan (2004) has observed that teachers often only stick to very low-level mathematics concepts,

> Unfortunately, in the absence for many teachers of sufficient knowledge, or interest in learning more, children are exposed to their teacher's reinforcement of lower level concepts such as colour, counting and shape, even when such reinforcement is inappropriate or unnecessary. (p. 99)

These mathematical ideas of codes and secret numbers lasted for many weeks. Emma added a variety of safes, secret boxes and door codes to extend the children's thinking. One of the main features of her practice is that she carefully listens to the children and takes her cues from them. She has developed an intuitive awareness of children's meaning making (Carruthers, in progress).

The teacher Emma is a *change agent* (Rodd, 2015), and I argue that it is professional colleagues like Emma who, on the ground, will change practice; they are the mediators and enablers of authentic mathematical play practices that unlock the children's mathematical thinking. The pedagogy of early mathematics depicted in this vignette is not found in a list of tips for teachers, but is much deeper than this, respecting teachers' professionalism and children's intellectualism. The pedagogy of young children's mathematics can only flourish in a democratic nursery culture, with teachers who understand the context and cultures of the young children's worlds they teach; who can facilitate play opportunities collaborating and co-constructing alternative play worlds; have an awareness and knowledge of the breadth of mathematical learning and who have the passion to want to delve further. This will support young children's higher-level mathematical thinking.

Commentary 3: A perspective from Denmark

Authors: Lone Hattingh

Whilst early childhood education in Denmark is often held in high regard with its recognition of play and social pedagogy as a fundamental feature of education, there is nonetheless a move towards a curriculum with defined goals or themes (Broström, 2017). Brogaard Clausen (2015) makes reference to schoolification of the early childhood curriculum in England, yet the impact of evaluation and standardisation in response to international comparisons can be seen in a move to a more prescriptive curriculum in the Nordic countries too (Jensen, 2014). Broström further states that the Danish early years curriculum has changed to include references to six themes which include goals and developing concepts, and to preparation for school. This can be seen as part of a discourse about outcomes, performance and the economic benefit of early childhood, with the view that children are an investment for the economic future of the country (Jensen, 2014).

However, principles of social pedagogy and democracy continue to underpin Danish preschool practice. Childcare is under the direction of the Ministry for Children and Social Affairs (2018) which publishes the curriculum for early years. The curriculum is made up of six broad learning themes and was originally introduced in Denmark in 2004 in response to criticism that there was too much free play, and that the adults working with children should have a more systematic and organised approach to their learning (Ahrenkiel, 2015). There is a focus on children's developing understandings of number and mathematical concepts in the theme for nature and science (Ministry for Children and Social Affairs, 2018), with the environment providing opportunities for developing concepts such as those of quantity, size, position, weight. This learning theme encourages pedagogues to arrange the environment so that children have opportunities to experiment with number, area and form in their early experiences of mathematical language and thinking. This resonates with the way in which the children's teacher Emma provides multimodal resources for the development of their play and mathematical thinking. Within the Danish's curriculum's six learning themes, play has a strong and visible presence, and guidance is broad. This allows settings to interpret them as they consider relevant.

Klitmøller and Sommer (2015) argue that the impact of neoliberal values, whilst less visible in Denmark than in many other countries, result in separating children's learning into separate themes. Citing George Monbiot, Moss (2019) explains that neoliberalism has become the dominant "powerful political narrative" of our time (p. 16), emphasising competition, economics and individual choice. This has brought about a discourse which is based on standardisation and outcomes. Consequently, neoliberalism has encouraged simplistic forms of evaluation and assessment which are easy to measure and therefore favoured by some governments. Neoliberalism can be seen to contradict the values of the Nordic social pedagogical approach to early years education which foreground holistic

learning, education and play (Kragh-Müller, 2017). Broström (2017) suggests that the relationship between play and learning is at times tenuous and calls for a synthesis where the "learning dimension is embedded in play" (p. 13). This embeddedness can be seen in Jayden and Isaac's play with the wooden building blocks which they then transported to a more suitable place as they developed their imaginative play. The materials were available to the children to use spontaneously in their play: wooden building blocks, furniture (a small cupboard), keyboard, clip board, tape and mark making implements. In his seminal work *Before Writing,* Kress (1997) suggests that materials need to be "to hand" so that children can select those that represent their thinking and their meanings.

The materiality of play

According to Broström it is the meaningful elements of play that facilitate children's learning. The accessibility of the materials enriched Jayden and Isaac's play and enabled them to take a multimodal approach to create a situation which reflected their own interests and their cultural understanding about the purpose, function and usefulness of their safe. Drawing on Bishop's (1998a) "six fundamental activities", the multimodal elements of Jason and Isaac's play provide opportunities for the development of their mathematical learning in a meaningful context. In developing their thinking, Jayden and Isaac used more than language and the available materials; they used sounds (beep, beep, beep) and actions, such as tapping on the keyboard and pressing buttons for access to the safe. The complexity of their play is evidence of their growing mathematical understanding while at the same time moving away from the binary either/or of play and learning (Broström, 2017).

Of importance for Lenz Taguchi (2010) are time and space, which are evident as the children develop their ideas and draw their teacher Emma in to their play. In the vignette, the children's play with their safe takes place over several days, reflecting their teacher Emma's calm approach in encouraging the children to develop their ideas in their own time. Within this unhurried environment, they are able to explore their ideas in depth, showing that they are resourceful and capable of complex thinking. Pacini-Ketchabaw, Kind and Kocher (2017) explain how time "to slow down, to listen" provides children with opportunities to discover the "thingness" of materials around them and to develop their thinking (p. 9). In this way this window onto the children's play makes visible their "intra-active relationship" (Lenz Taguchi, 2010, p. 10) with each other, their teacher, the materials as well as the time and space which is freely available to them.

Jayden's wavy lines on the paper and clipboard represent the result of the block being taken away by another child: yet while this line may not represent numerals in the traditional sense, his understanding of its use in representing subtraction is evident in the way he responds and makes this part of his play. It is frequently the apparently unremarkable that goes unnoticed, suggesting that careful and close engagement (or entanglement, as Lenz Taguchi might suggest) with the data is needed to try to understand the development of the

child's thinking. The different ways in which Isaac and Jayden made use of the materials also reflect their inventiveness and resourcefulness, confirming the deep learning that takes place in their pretend play scenario. This scenario would sit comfortably within the Danish learning themes (Ministry for Children and Social Affairs, 2018) where the documentation begins with a declaration of the importance of play and a broad understanding of learning and community.

Shared voices: reflections on playing, learning and mathematics

Bishop (1988a) argues that mathematics is more than "meaningless pushing around of figures which many children still unfortunately experience" (p. 186). As this vignette shows, high quality and sustained impromptu pretend play (Bishop's 'imagined reality') can also support children's conceptual learning such as mathematics.

The authors of this chapter – from England and Denmark - value children's self-initiated and spontaneous play not as an 'activity' to be *used by teachers* for teaching mathematics, but for the opportunities, experiences and qualities it engenders in young children. In this nursery school children draw on many modes to explore, make and communicate their meanings, and in this, the nursery school's practice differs from much adult-led mathematical pedagogy in England. Whist the early childhood curriculum in Denmark (Ministry for Children and Social Affairs, 2018) is made up of six themes: personal development; social development; communication and language; body, senses and movement; nature, outdoor experience and science; culture, aesthetics, and community (author's translation), the emphasis on play favours a holistic approach in which specific elements of learning are loosely defined. This contrasts to the English curriculum where mathematics appears as a distinct area within the theme for "learning and development" (DfE, 2013).

But for many in early years education there has been an apparent division between play and learning, Broström (2017) considering an "either/or" positioning unproductive, a view that denies a unified concept of play and learning (p. 1). Such a perspective endorses a distinct divide between the two where in today's world, "learning" belongs in the teacher's domain and "play" in the child's, pretend play often narrowly circumscribed. In this vignette of Isaac and Jayden's play, their mathematical learning takes place within a context of deep cultural meaning for them, alongside much other learning which is not so easily described as they discuss and collaborate with each other and their teacher Emma. One might argue that the requirement to record children's learning in narrowly defined ways as required by the English *Early Years Foundation Stage* (DfE, 2013), fails to acknowledge other learning that is taking place alongside and within this potentially rich play experience.

Broström (2017) argues that "children learn through interaction and communication *when they experience the activity as meaningful* and when it sparks their creativity and imagination" (p. 7, emphasis added), concluding that we can "overcome this educational conflict (play versus learning) and identify a shared understanding where the learning

dimension is embedded in play" (p. 11). Thus, in reflecting on Isaac and Jayden it is evident that their play fulfils all these criteria, suggesting that the children's experiences of play and learning, expressing the nursery school's shared democratic values and philosophy and creating something of Broström's "unified theory" of play and learning.

The children's experiences are a reflection of the high quality of teaching in this nursery school, underpinned by its philosophy and research culture, and is clearly a setting which recognises that children have a relationship with their social and material surroundings, in a "state of interdependence" (Lenz Taguchi, 2010, p. 51). With the emphasis on children's own imaginative play, the in-between spaces also have meaning. These children attend a nursery setting which encourages free and uninterrupted exploration in their play, enabling teachers to observe and assess children's learning and experiences through a wide lens without the need to rely on developmentally defined outcomes.

Whilst Denmark seems to have put play at the forefront of learning in their curriculum document, this was also true in the first early years curriculum documents in England (DfEE, 2000) which offered hope for a greater understanding of play and making it officially recognised; but interpretations became blurred and through the years it has become more the dictate of Ofsted and government regulations (Ofsted, 2017). Like Denmark, play in this nursery school has "a strong and visible presence", and the vignette of Isaac and Jayden "would sit comfortably" there, but to what extent is this always true for England?

Oberhuemer (2005) calls for "democratic professionalism" where early years teachers and practitioners need to see themselves as "interpreters and not mere implementers of curricular frameworks" (p. 13). From each of the commentaries it was clear that the teacher in the vignette characterised democratic professionalism; but teachers like Emma are perhaps atypical, at least, in England. Perhaps therefore, all professional development for early childhood teachers and practitioners needs to consider emphasising professional identities, critical reflection and teachers as agents of their own learning (Rodd, 2015).

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Notes

[i] Nursery Schools in England are a unique early years' provision that are government funded. They have a tradition dating back to the nineteen twenties, rooted in the work of Margaret Macmillan and Susan Isaacs (Giardiello, 2013).

[ii] Both teachers and early years' practitioners work in the nursery school, but for brevity the word "teacher" is used throughout.

[iii] Regrettably, Isaac's mathematical inscription is not sufficiently clear to re-produce here.

[iv] The nursery's approach to mathematics developed from work by Carruthers and Worthington.

[v] Prior to four years of age some children will attend nursery schools whilst others may attend pre-school playgroups or childminders.

[vi] The teaching of 'synthetic phonics' especially, has had a significant impact on all aspects of teaching and learning in the Foundation Stage in England. In turn this has exerted new pressures on early years mathematics.

[vii] The four rules listed here and the explanation of each are taken from van Oers (2013, pp. 191-192).
In this instance, technology appears to be a dominant interest (see for example, Pirani and Hussain (2019) who explore the value of technology for learning in the early years).

[viii] The term "hot spot" relates to biodiversity and describes a region that is a significant reservoir of biodiverse species. The diversity (of children's mathematical thinking, talk, conceptual knowledge and graphical representations) are exemplified in this chapter and in chapter 2, thereby endorsing it as a *mathematical* (and play) hot spot.

[ix] The title, "National Teaching School" is awarded to a school having previously developed research and is a centre of initial teacher training: these school are funded for their work.